Curriculum Guidelines for the 12-Year Basic Education

Elementary School, Junior High School, and Upper Secondary School

The Domain of Mathematics

Ministry of Education

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I. Fundamental Beliefs

The Curriculum Guidelines of the 12-Year Basic Education are developed based on the spirit of holistic education while embracing being self-initiative, interactive and seeking the common good as basic beliefs, and adopting the vision of enabling each child to be successful, both in terms of nurturing individual potential and lifelong learning. The Curriculum Guidelines of the domain of Mathematics resonate basically with the ideas and visions of the General Guidelines. They start by regarding mathematics as a kind of language, a kind of practical science of pattern and as a kind of humanistic literacy. The curriculum design is developed by taking these features into consideration. It aims at providing every student an opportunity to learn and cultivate a literacy in appropriately using calculators and other instruments. These beliefs are explained below.

1. Mathematics as a form of language; mathematics can be introduced with reference to some tactics from language learning

Language plays a crucial role in the development of civilization. Development of mathematics is immersed within the development of the natural language of everyday experiences. Some examples include expressions concerning quantities and shapes, as well as descriptions about their relationships. These are concepts in mathematics but also part of the vocabulary that are commonly used in daily life. While words are used for everyday communication, mathematics as a specific form of language can be used to explain some phenomena in a more concise and precise manner.

Since mathematical expressions are concise in conveying meaning, they can be used to explain various complex phenomena in terms of simple and clear formulae and theory, thereby enabling complex matters to appear simpler. Since mathematical expressions are precise in conveying meaning, they can fill the gaps that are left when natural language is insufficient for conveying the ideas. Furthermore, mathematics is effective for nurturing algorithmic ability, logical reasoning and abstract thinking.

In accordance with the above characteristics, mathematics instruction should try to retain those effective aspects of language learning by introducing mathematical concepts first through manipulation and explanation with concrete examples. After students have gained enough understanding of the concepts and computational rules, they can be introduced to the abstract theory behind those concepts.

2. Mathematics as a science of pattern with practical values in different disciplines; mathematics instruction should pay attention to include interdisciplinary contents

Mathematics has been extensively applied to meet our daily needs, enabling us to explore the mysteries of nature, interpret social phenomena, analyze economic and financial matters, and support the development of sciences and technology. These fields may look complicated on the surface. Yet, we can usually discern deep-rooted and invariant patterns through applications of mathematical analyses. Mathematics is a discipline that is very useful for studying patterns. There are many practical applications of mathematics. For instance, proportion can be used to calculate the exchange rates among different currencies and the dilution of concentrations of various solutions. The properties of prime numbers can be used to develop cryptographic systems to greatly enhance security in information transfer. Exponential rules can be applied to compute compound interests from saving deposits, growth rates of organisms, and the half-lives of radioactive elements. Trigonometry is used in measurement and trigonometric functions can be used to study waves (such as sound, light and water waves). Finally, statistics is used to analyze big data and make predictions despite uncertainties in the real world. As mathematics can be applied in many disciplines, instruction of mathematics should pay attention to include interdisciplinary contents.

3. Mathematics as a kind of humanistic literacy; mathematics education should nurture students' appreciation of the beauty of mathematics

Mathematics has its internal locus of development. After being applied to conduct investigations, make inductions, solve conjectures and construct arguments for thousands of years, mathematics has become a form of language through which mankind can discourse with nature. Furthermore, mathematics has been responding to the needs of our society to solve a variety of problems, it has thus played an indispensable role in the advancement of civilization. Different cultures, however, have established different approaches to do mathematics. For examples, Eastern mathematics in ancient times emphasized iconic type of inductive reasoning, while Western mathematics emphasized abstract type of deductive reasoning. The history of mathematics can help us understand these differences regarding how different cultures at different eras have taken different courses of adventures in mathematics. It can also help mathematics educators clarify the major topics and contexts for mathematics instruction. Thus if educators can assimilate mathematics history into their instruction whenever appropriate,

such efforts can enhance both the quality of mathematics instruction as well as the learning achievement of students. By recognizing the cultural aspect in mathematics, not only can students elevate their learning of mathematics from instrumental understanding to relational understanding, they can also appreciate more the humanistic value of mathematical knowledge. The mathematics curriculum will then have a better chance of achieving the educational goals of “adaptive education to develop talent” and "lifelong learning.”

4. Mathematics education should provide every student an opportunity for sense-making of the learning contents

Mathematics differs from many other disciplines in that its knowledge structures accumulate in logical orders, so that if a student has not mastered a certain concept at a previous stage, he or she will encounter difficulty at a later stage. Moreover, the learning of mathematics requires both intuition and sound reasoning. Students of the same age, however, demonstrate individual differences in their understanding of mathematics. As a result, curriculum compilation can only satisfy the majority of the students. As regards the implementation of the curriculum, it should be carried out by means of individualized instruction, scaffolding and well-organized sets of learning activities. The goal is to provide each student an opportunity for sense-making of the learning materials in each class. For those who are slower in learning, mathematics educators can relax their pace of instruction and focus their coverage on the basic concepts of the curriculum. For those who are capable of accelerated learning, mathematics educators can design deeper and wider curriculum or inquiry activities on special topics, thereby inspiring students’ motivation to learn. For those who fall behind in their learning, mathematics educators can design, in a timely manner, remedial lessons in accordance with such factors as students’ readiness and learning styles. Special efforts should be invested in applying these remedial strategies in classrooms as a way of providing adaptive guidance to those in need.

5. Mathematics instruction should nurture students towards a literacy of correct use of mathematical instruments

Mathematical instruments are of great help to mathematics instruction. Besides those traditional teaching aids such as compasses, set squares, graph paper, etc., instruments for the information era including calculators, computers, networks, multimedia, and mobile tools are all useful learning tools. Our country, however, lags far behind other developed countries even in the

instruction of the use of calculators. As such, the present revision of the curriculum guidelines places special emphasis on the effective use of computing instruments. Instruction in this aspect should start with calculators, and then guide students towards more advanced computational tools, including spreadsheets and mathematical software. Mathematics is a kind of science of pattern, which can be explored with the assistance of calculators and computers. However, there are floating errors in computations by calculators. Instructors should remind students regarding when to use calculators and their limitations, thereby cultivating a correct attitude towards their usage. For example, after students have learned the principles behind computation, they can use calculators to take care of tedious calculations so as not to hinder their learning efficiency. They can use calculators and other devices to do tedious calculations, handle statistical data as well as calculations involving exponential, logarithic and trigonometric functions. It is suggested that calculators can be used starting from Stage 4 (that is, junior high school). Mathematics instructors can also include the use of computers for instruction whenever appropriate.

II. Curriculum Goals

Now that we are in the 21st century, mathematics continues to take big strides in applications in many situations. Areas such as the sciences, technology, information and finance make increasing demands for talents with training in mathematics and the sciences. This curriculum is compiled in accordance with the abovementioned beliefs as well as consideration about possible changes in the near future, including students' potential career planning, national course of economic development, and participation in international events. It is the intention of this curriculum to provide twelve years of high-quality basic training in mathematics that prepares students with enough background to pursue further training in universities or enter the workforce. One of the main focuses of schools is to help students learn knowledge and skills that are useful in their careers later on. The nature of mathematics is abstract. Yet this feature has endowed mathematics with a deep level and a broad aspect of applications. Thus, an important goal of the curriculum is to find out how to help students of different ages, and with different abilities, interests and focuses to attain mathematical literacy pertaining to both theoretical and applied aspects of mathematics. Mathematics education should kindle in students their learning motivation and curiosity, together with fostering their abilities to inquire, reason, discern and act. It should cultivate in students a willingness to learn as well as to engage in mathematics activities in an

active and persistent manner. It is envisioned that students, experiencing joy in learning and a sense of enhanced self-worth, will have their potential for life stimulated and hence commence and continue on their journey to a balanced and wholesome personal development.

In order to achieve these visions, mathematics education should provide enough opportunities to students to learn by allowing for the following curriculum goals:

1. Provide students with customized opportunities to learn and nurture in them a positive attitude as well as self-confidence in partaking in mathematical inquiries.
2. Nurture in students their curiosity and their abilities in observing patterns, computation, abstraction, reasoning, communication and expressing ideas mathematically.
3. Nurture in students the correct attitudes and proper manner in using mathematical instruments for carrying out algorithms and problem solving.
4. Nurture in students a disposition towards considering, analyzing and solving problems being encountered from a mathematical mindset.
5. Nurture in students a disposition towards applying mathematics to solve problems encountered in daily life and a willingness to learn the mathematical knowhow that are specifically required in other disciplines.
6. Nurture in students an appreciation for the potential in using mathematics to treat complex issues with simple formulae and an appreciation for the rigor and the beauty in mathematical structures.

III. Time Allocation

The time allocation and the scheme for the required or elective curriculum strands at each stage are listed as follows:

1. Elementary school from Grade 1 to Grade 6: 4 classes per week for a total of 160 minutes.
2. Junior high school from Grade 7 to Grade 9: 4 classes per week for a total of 160 minutes.
3. Upper secondary school Grade 10: Mathematics is an 8-credit required course for a total of 200 minutes per week.
4. Upper secondary school Grade 11: Mathematics is an 8-credit required course for a total of 200 minutes per week, with two alternatives Mathematics A and Mathematics B for selection.
5. Upper secondary school Grade 12: Broaden and deepen mathematics as an elective course with a total of 200 minutes per week, with two alternatives Mathematics I and Mathematics II. Students can choose either alternative for 8 credits maximum or they can decide not to pursue

any mathematics at this level.

IV. Core Competency

In order to implement the aforementioned beliefs and goals, the compilation of the mathematics curriculum is oriented with respect to the concept of core competency, which refers to the knowledge, skills and attitudes that an individual should possess when facing problems encountered in life as well as future challenges. The following table is the manifestation of the core competencies of the mathematics domain that caters to its fundamental beliefs and curriculum goals, and corresponds to the contents of the core competency outlined in the General Guidelines.

Core Competency Dimension	Core Competency Item	Item Description	Core Competencies of the domain of Mathematics		
			Elementary School (E)	Junior High School (J)	Upper Secondary School (U)
A **Autonomous Action**	**A1** **Physical and Mental Well-ness and Self-Ad-vancement**	Possess the ability to conduct sound physical and mental developments and maintain a proper view of human nature and self-image. Through decision-making, analyses, and knowledge acquired, students can effectively plan their career paths, search for meaning in life, and continually strive for personal growth.	數-E-A1 Possess an interest and a curious mind towards mathematics; possess an active attitude towards learning mathematics; and be able to apply mathematics language in daily life.	數-J-A1 Have confidence and a positive attitude towards learning mathematics; be able to use appropriate mathematics language to communicate; and be able to apply what is learned to solve problems in daily life	數-S-U-A1 Be able to persist in inquiry and solving mathematical problems, possess the ability to reason mathematically; be able to use mathematics language in communication that requires accuracy and rationality; have the ability to learn in order to fulfill one's own career plan and lifelong development.
	A2 **Logical Thinking and Problem Solving**	Possess the competency in systematic thinking to understand problems, engage in analyses, think critically, and endeavor in meta-thoughts, with the ability to reflect and take action, to effectively handle and solve problems in daily life.	數-E-A2 Possess the ability to carry out basic arithmetic operations; be able to identify basic forms and their relationships; and be able to use mathematics to express and solve problems encountered in daily life.	數-J-A2 Possess the ability to perform arithmetic operations involving rational numbers, radicals and the coordinate system; be able to use symbols to stand for numbers or geometrical objects and reason or compute accordingly; be able to analyze the nature of problems encountered in daily	數-S-U-A2 Possess the basic ability to formulate mathematical models and use them to solve realistic problems; understand the characteristics of mathematics in that deductive proofs have to be carried out to substantiate the claims induced based on observations and appreciate the value

				life or from a reasonable imaginary context and then solve them.	behind this practice.
	A3 Planning, Execution, Innovation, and Adaptation	Possess the ability to devise and execute plans, as well as the ability to explore and develop a variety of professional knowledge; enrich life experience and fully utilize creativity to improve one's adaptability to social changes.	數-E-A3 Be able to observe that problems in daily life can be related to mathematics; be willing to attempt and devise a plan to solve the problems encountered; and be able to transform the mathematical solutions back into daily life contexts.	數-J-A3 Possess the ability to identify the relationship between problems in daily life and mathematics; be able to formulate plans for problem solving from multiple and flexible perspectives; and be able to transform the mathematical solutions back into daily life contexts.	數-S-U-A3 Possess the ability to transform realistic problems into mathematical problems; be able to explore problems and then formulate and implement a plan to solve the problems; be able to solve mathematical problems from multiple, flexible and creative perspectives; and be able to transform the mathematical solutions back into daily life contexts.
B Communication and Interaction	**B1 Semiotics and Expression Semiotics and Expression**	Possess the ability to understand and use various types of symbols, including languages, characters, mathematics, bodily postures, and arts to communicate and interact with others, understand; and have empathy for others; and be able to make use of these abilities in daily life or at the workplace.	數-E-B1 Possess the ability to translate among verbal, numerical and symbolic representations; proficient in operations related to time as well as weights and measures; recognize geometrical forms and shapes encountered in daily life; be able to express formulae in symbols.	數-J-B1 Possess the ability to handle mathematical relations in geometry and algebra and use them to describe phenomena in contexts; be able to use mathematical language to describe basic properties and relations that exist in a plane or space based on learning experiences; able to determine the degree of uncertainty of events encountered in daily life using basic statistics and probability.	數-S-U-B1 Be competent in using mathematical symbols to describe states, relations and operations, and appreciate the value of supplementing daily language and mathematical language with each other; be able to execute the operations according to the symbols in order to solve problems from different contexts; be able to demonstrate the process of operation or argumentation.
	B2 Information Technology Literacy and Media Literacy	Possess the ability to effectively use technology, information, and media of all types; develop competencies related to ethics and media literacy; and develop the ability to analyze, speculate about, and criticize	數-E-B2 Possess the ability to read off and produce simple statistical graphs and tables.	數-J-B2 Possess the competence to use calculators to enhance learning effectiveness, including an understanding about their applicability and limitation; and realize the value of supplementing mathematics	數-S-U-B2 Possess the competence to use calculators and computer software to enhance learning effectiveness, including an understanding about their applicability and limitation, and realize the value of supplementing

		humans' relationships with technology, information, and media.		knowledge and mathematical instruments with each other, and can use them to carry out mathematical procedures; recognize the basic characteristics of statistical data.	mathematics knowledge and mathematical instruments with each other, and can use them to carry out mathematical procedures; able to interpret, criticize and reflect on the nature of the information and issues presented in the media.
	B3 Artistic Appreciation and Aesthetic Literacy	Possess the abilities of art awareness, creation, and appreciation, experience artistic culture through reflection on arts in daily life, enrich artistic experiences, and develop the ability to appreciate, create, and share arts.	數-E-B3 Possess the ability to notice mathematical forms and shapes or patterns in artistic works.	數-J-B3 Possess the ability to identify geometrical forms and shapes or quantitative relationship in artistic works; and appreciate the beauty of mathematics in mathematical derivations.	數-S-U-B3 Possess the competence to realize that mathematics can serve as a principle for artistic creation and as a model to describe human perception; demonstrate willingness to explore whether mathematics principles can be applied to assist in artistic creation.
C **Social Participation**	**C1 Moral Praxis and Citizenship**	Possess competency in putting morality in practice from the personal sphere to the social sphere, and gradually develop a sense of social responsibility and civic consciousness; take the initiative in concern for public topics and actively participate in community events; pay attention to the sustainable development of humanity and the natural environment; and exhibit the qualities of moral character to recognize,	數-E-C1 Possess the attitude to base discussions on evidence and communicate in an organized manner.	數-E-C1 Possess the attitude to base discussions on evidence and provide reasonable arguments; and be able to communicate rationally and collaborate with others.	數-E-C1 Possess the attitude to base discussions on evidence and build reasonable arguments; develop the competency to communicate rationally with others; become reflective and ethical citizens.

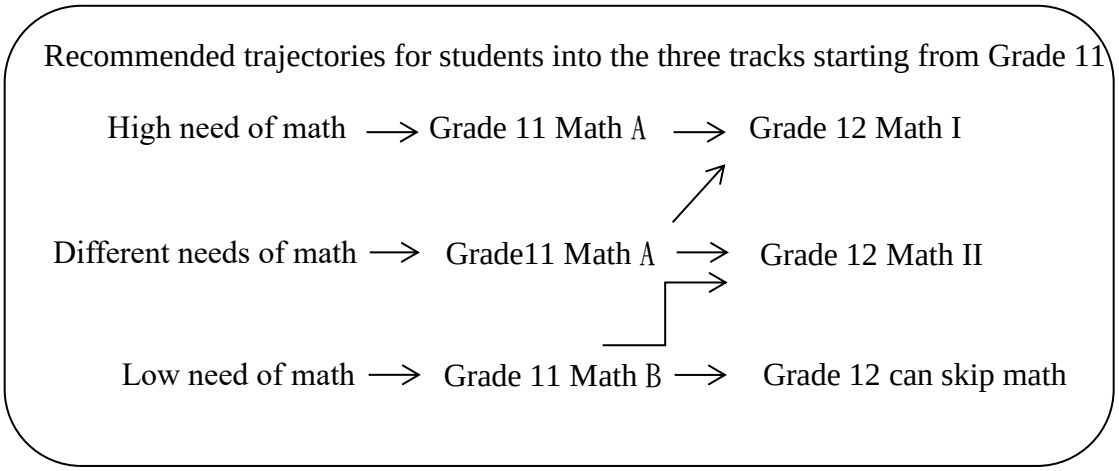
		appreciate, and practice good deeds.			
	C2 Interper-sonal Rela-tionships and Teamwork	Possess the competency in exhibiting friendly interpersonal feelings and the ability to establish strong interactive relationships; establish communication channels with others, tolerate outsiders, and participate and serve in social activities and other activities requiring teamwork.	數-E-C2 Ready to cooperate with others to solve problems and pay due respect to different approaches in solving problems.	數-J-C2 Ready to interact and communicate well with others to solve problems; pay due respect to different approaches in solving problems.	數-S-U-C2 Possess the ability to cooperate with others to solve problems; pay due respect to different approaches in solving problems; possess the ability to build good interactive relationships with others.
	C3 Multi-cul-tural and Global Un-derstanding	Stick to one's own cultural identity, respect and appreciate multicultural¬ism, show active concern for global issues and international situations, demonstrate the ability to adapt to the contemporary world and to social needs, develop international understanding and a multicultural value system, and strive for world peace.	數-E-C3 Possess the competence to understand and care for the multicultural and the language aspects of mathematical representations, and be able to compare with those from one's own culture.	數-J-C3 Possess the competence to observe and accept, with a global perspective, the historical and geographical backgrounds in the development of mathematics.	數-S-U-C3 Possess the competence to appreciate, with a cross-cultural perspective, the historical and geographical backgrounds in the development of mathematical concepts and tools; understand that the development of mathematics has spurred the development of technology; and recognize examples pertaining to cultural differences in the development of mathematics.

V. Learning Focus

Learning focus has two dimensions, namely, learning performance and learning content, and its purpose is to guide the design of curriculum, compilations of instructional materials, reviews of textbooks and assessments of learning. Classroom instruction should be implemented in accordance with the specifications detailed in the learning focus, which are based on the nature of the subject matter and by observing the ideals and goals mentioned above. Furthermore, the specifications are in alignment with the core competencies discussed in Section IV above, so that the actual curriculum that manifests the content of the learning focus will correspond closely to

the core competencies.

There are three tracks of mathematics courses for students to pursue starting from Grade 11 in the upper secondary school. For students with high need of mathematics, they can study Mathematics A in Grade 11 followed by Mathematics I in Grade 12. For students with different dimensions of need of mathematics, they can study Mathematics A in Grade 11 followed by either Mathematics I or Mathematics II in Grade 12. For students with less need of mathematics, they can choose to study just Mathematics B in Grade 11. Given that upper secondary school students do not easily find their interest too early, the mathematics curriculum is designed in such a way that switching tracks is not very difficult. Students who take Mathematics B in Grade 11 will have the opportunity to acquire the prerequisite knowledge so that they can enroll in Mathematics II in Grade 12. The following diagram shows the learning trajectories for students according to the three tracks of courses starting from Grade 11.



The compilation of the learning focus is according to the following categories: elementary school, junior high school, required courses for upper secondary school (subdivided into A and B versions for Grade 11), and deepen and broaden elective courses for upper secondary school (subdivided into I and II for Grade 12). However, its development is based on the teaching objectives of the following five learning stages.

Stage I (Grades 1 and 2): Students begin to master the concepts of number, quantity and shape. The emphasis is on students being able to understand natural numbers and their operations; recognize the concept of length and simple shapes.

Stage II (Grades 3 and 4): With regard to number, students are expected to accurately grasp the four fundamental operations of arithmetic and mixed operations. They should begin to cultivate

good number sense and learn the concepts of fractions and decimal numbers. As for quantity, length will be used as the basic concept and students are expected to handle common units and their computation. As for geometry, students should develop the ability to understand geometric figures through angles and sides, as well as the properties of geometric figures.

Stage III (Grades 5 and 6): Students are expected to accurately grasp the four fundamental arithmetic operations of fractions and decimals. They should be able to apply commonly seen numerical relations to solve problems encountered in daily life. They can recognize geometrical properties of simple plane and solid forms and shapes and know how to compute their areas and volumes. They should be able to create simple statistical charts.

Stage IV (Grades 7 to 9 of junior high school): With regard to number, students are expected to understand the concepts of negative numbers and surds, as well as their computations. They should also understand the meaning of coordinates and what they represent. As for algebra, students should be proficient in computations involving algebraic expressions, solving equations and understanding the concept of simple functions. As for plane geometry, students in different grades should learn geometry in a progressive manner, first by way of intuition (through observing, identifying, and describing), followed by using measurements and then through reasoning. Space geometry is scheduled to be learned at a later phase. In addition, students should understand the meaning of basic concepts in statistics and probability, as well as recognize basic statistical methods.

Stage V (Grades 10 to 12 of upper secondary school): With regard to number, students are expected to have a unified understanding of the real numbers. They should have further understanding of the counting principle and be able to apply the principle in practice. For students who enroll in either Mathematics A or Mathematics B, they are expected to extend their understanding of numbers to include complex numbers. In particular, students who take Mathematics A should also understand the geometrical meaning of complex numbers. As for geometry, all students can have the opportunity to study the basic concept of space by way of coordinate geometry, with which they can realize a connection between geometry and algebra. They should be able to recognize the basics of linear algebra. In particular, students who take Mathematics A are expected to be proficient in the operations of vectors in space, so that they can pursue further in the areas of coordinate geometry and linear algebra. With respect to function, all students can have the opportunity to recognize three types of basic functions: polynomial functions, logarithmic and exponential functions, and trigonometric functions. Students should be able to

discern the characteristics of their graphs and use these functions as mathematical models to solve typical problems. Students who take either Mathematics I or Mathematics II should be able to extend their understanding of functions to study the essentials of calculus, and be able to apply their knowledge to solve basic problems in their respective fields, namely, science and technology or business and management. With respect to uncertainty, all students should be able to describe data by basic statistics. They should also be able to use probabilistic and statistical reasoning to account for the degree of uncertainty in each given set of information. Students who take either Mathematics I or Mathematics II should be able to further understand the concept of a random variable and its distribution. In particular, students who enroll in Mathematics I should understand the concept of the geometric distribution.

1. Learning Performance

Learning performances are formulated from a student-centered perspective, paying attention to students' cognition (knowledge seeking, application and reasoning), affective domain (appreciation and attitude) and the manifestation of their learning through applications in daily life. The consideration here includes both the content and content-free dimensions, and directly manifesting or at least correspond to core competencies of the General Guidelines. These dimensions have been restated in easy-to-understand terms for mathematics instructors and the public rather than using descriptions from educational theories. Mathematical performances are expressed using some of the cognitive terms that serve as a description of the learning progress expected from students. The meanings of some of the special terms are explained as follows:

(1) **Recognize, Understand, Proficient:** "Recognize" includes to be aware, to know and to identify. "Understand" includes to discern, to make connections among concepts, and to comprehend. "Proficient" includes being able to apply and solve problems, to reason and to be proficient in procedural type of content. If a mathematical concept can be taught within one learning stage, then the learning performance to be expected will be presented using descriptors that denote the more mature learning status. For example, if the term "understand" appears in a certain entry while the term "recognize" is not mentioned, it means either recognition is assumed to have been achieved or that recognition and

understanding have to be achieved within the same stage.

(2) **Context or Situation:** Students often need to relate to a certain context of experiences when they try to understand some mathematical concepts or patterns or to solve problems. Not only are the contexts of experiences very helpful in learning, they can also facilitate future applications of knowledge in similar contexts. There are two types of contexts frequently mentioned in these curriculum guidelines. The first type refers to situations where the experiences are encountered, such as daily situations and concrete situations (see the next paragraph). The other type refers to the contexts of experiences that cater for learning core type of mathematics concepts, such as the situation for equal sharing and the situation for measurement.

(3) **Concrete situations:** Usually, when students learn mathematics, appropriate examples or applications from their daily experiences need to be provided as hints to guide them to learn. These experiences are generally called concrete or specific situations (corresponding to "recognize" and "understand"). For Stage I and Stage II in elementary schools, the concrete situations and the daily situations are basically indistinguishable. However, starting from Stage III, as students become more familiar with mathematical concepts, representations (such as the array model of multiplication) or even abstract thinking experiences (such as number sense), the concrete situations from which students can learn need not be limited to just daily situations. For example, for students in Grades 5 and 6 when they learn factors, multiples or prime numbers, the most appropriate concrete situation is not problems from everyday life but their familiarity of the properties of whole numbers. From Stage IV onward, the concrete situations can even include mathematical situations or parts of theoretical contexts from other fields.

(4) **Problem solving:** In the curriculum guidelines, mathematical problem solving generally refers to the application of mathematical concepts and procedures to solve everyday, mathematical and other application problems. The problem-solving process includes understanding the meaning of the problem, choosing a feasible strategy, transforming the strategy into a mathematical problem, using mathematical knowledge to solve the mathematical problem, testing out and interpreting the meaning of the solution, and determining whether the solution can

fulfill the problem solving requirements. However, reflection, generalization and communication that are more demanding are not required in these curriculum guidelines.

(5) **Manipulative activities:** Manipulative activities generally refer to activities that require observation, concept formation and even making simple connections among various concepts. For Stage I and Stage II in elementary schools, students are in the beginning period of establishing various basic concepts and thus lack enough mathematical experiences. As a result, their learning must be guided through activities related to the context of daily life. Therefore, the teaching of many topics should start with operative activities.

(6) **Reading off:** Reading off (or just reading) refers to the reading of information, including the correct understanding of the ways data are presented (tables, statistical graphs), and being able to answer direct questions and simple extended questions about the data given (such as questions connected to other mathematical concepts). Problems that require more mature reasoning skills do not belong the area of "reading off. "

The contents of the learning performances are written according to the learning stages. The coding system is as follows.

The first code represents the "area of study, " which is denoted in lower case letters with **n** (number and quantity), **s** (space and shape), **g** (coordinate geometry), **r** (relationship), **a** (algebra), **f** (function) and **d** (data and uncertainty). The code **r** is adopted exclusively for elementary schools, which will be switched to **a** and **f** for junior and upper secondary schools.

The second code is the "learning stage, " which is denoted by Roman numerals in ascending order with I (Grades 1 and 2), II (Grades 3 and 4), III (Grades 5 and 6), IV (Grades 7, 8 and 9) and V (Grades 10, 11 and 12).

The third code is the serial number. It is not necessary for textbooks to compile the learning contents according to the serial number at any given learning stage.

Learning performances are shown in the two tables below. The first table is sorted according to the learning stages. For convenience of comparison, all contents for the same area of study have been

assembled together and presented in the second table. The entries there are sorted according to the second code for the learning stages.

Learning performance organized according to the learning stages

Code	Learning Performance (arranged according to the Learning Stages)
Stage I	
n-I-1	Understand the place value structure for numbers within one thousand, and be able to use it as the foundation for the four fundamental arithmetic operations.
n-I-2	Understand the meaning of addition and subtraction, and be proficient and fluent in their computation.
n-I-3	Apply addition and subtraction through computation and estimation to solve application problems encountered in daily life.
n-I-4	Understand the meaning of multiplication; be proficient in ten-by-ten (10×10) multiplication; and begin to be engaged in activities related to partitive and quotative division.
n-I-5	Solve simple two-step application problems in specific contexts.
n-I-6	Recognize unit fractions.
n-I-7	Understand the concept of length and its common units; perform measurement, estimation and computation.
n-I-8	Recognize the concepts of capacity, weight and area.
n-I-9	Recognize the concepts of moment and duration of time together with their common units.
s-I-1	Attain initial recognition of the geometrical properties of objects and common geometrical forms through manipulative activities.
r-I-1	Learn symbols of operations, relational symbols and the convention of arithmetic expression as part of the mathematical language.
r-I-2	Recognize the operation rules of addition and multiplication.
r-I-3	Recognize the inverse relationship between addition and subtraction and apply it to solve problems.
d-I-1	Recognize different modes of classification, able to actively collect, classify and make simple records and explanation.
Stage II	
n-II-1	Understand the place value structure for numbers within a hundred million, and be able to use it as the foundation for various computations and estimations.
n-II-2	Be proficient in adding, subtracting and multiplying relatively larger numbers through computation or estimation, and be able to apply them to solve problems in daily contexts.
n-II-3	Understand the meaning of division, be able to compute and estimate, and be able to apply them to solve problems in daily contexts.
n-II-4	Solve estimation problems involving the four arithmetic operations encountered in daily contexts.
n-II-5	Solve two-step application problems in daily contexts.

Code	Learning Performance (arranged according to the Learning Stages)
n-II-6	Understand the meaning of fractions with the same denominator together with the addition, subtraction and integral multiples of such fractions, and be able to compute and apply them. Recognize the meaning of equivalent fractions and relate it to simple comparisons between fractions with different denominators as well as the meaning of adding and subtracting them.
n-II-7	Understand the meaning of decimals and the place value structure, be able to perform addition, subtraction and multiplication in the vertical format, and apply them to solve problems.
n-II-8	Be able to mark off whole numbers, fractions, and decimals on the number line, and compare and perform addition and subtraction. Understand that whole numbers, fractions and decimals are all numbers.
n-II-9	Understand the common units of and conversion involving length, angle, area, capacity and weight.
n-II-10	Understand the principles of addition and subtraction involving time, and be able to apply them to solve problems in daily contexts.
s-II-1	Understand the formulae of area and perimeter for the square and the rectangle and their application.
s-II-2	Recognize the meaning of congruence of plane figures.
s-II-3	Recognize commonly seen triangles, quadrilaterals and circles through their composing constituents.
s-II-4	Recognize the application of geometric concepts such as rotated angles, nets and solid figures through learning activities.
r-II-1	Understand the inverse properties of multiplication and division, and be able to apply them and solve problems.
r-II-2	Recognize one-dimensional and two-dimensional quantitative models and be able to explain and make simple inferences.
r-II-3	Understand the convention of combining the parts of a two-step problem into a single expression, and the convention of carrying out computations involving the four arithmetic operations.
r-II-4	Recognize the rules regarding addition and subtraction in a two-step computation problem, the rules regarding multiplication and division (for example, the order of multiplication and division is inconsequential in an expression involving only these operations), and be able to apply these rules to solve problems.
r-II-5	Understand mathematical expressions in verbal format.
d-II-1	Read off and produce one- and two-dimension tables and bar-charts, read off from line graphs and make simple inferences.
Stage III	
n-III-1	Understand the place value structure of the decimal system, and be able to extend this understanding to very large and very small numbers.
n-III-2	Solve common application problems requiring more than three steps in real life contexts.
n-III-3	Recognize the meanings of factor, multiple, prime number, the greatest common factor and the least common multiple, along with their computation and application.
n-III-4	Understand the meaning of reducing a fraction, expanding a fraction and taking common denominator, and be able to apply these concepts to perform the addition and subtraction of fractions with different denominators.
n-III-5	Understand the meaning of the fractional representation of the division of whole numbers.

Code	Learning Performance (arranged according to the Learning Stages)
n-III-6	Understand the meaning of fraction multiplication and division, also their computation and application.
n-III-7	Understand the meaning of multiplication and division involving decimals, and be able to perform computations in the vertical format and apply them to solve problems.
n-III-8	Understand how to take approximate numbers by rounding and carrying out realistic estimations.
n-III-9	Understand the meaning of a proportional relation, and be able to utilize it to observe, express, compute and solve problems involving rate, proportional scale, speed and baseline amounts.
n-III-10	Explore how to formulate the relationship between quantities in more complex contexts or models into correct mathematical expressions to make inferences or solve problems.
n-III-11	Recognize the common units and the conversion of quantities in relation to area, and be able to solve relevant application problems.
n-III-12	Understand the relationship between capacity and volume, and be able to apply this relationship in problem solving.
s-III-1	Understand how to compute the area of triangles, parallelograms and trapeziums.
s-III-2	Recognize the meaning of the ratio of the circumference of a circle to its diameter and understand how to compute the area of a circle and its circumference, and the area of a sector and its arc length.
s-III-3	Understand the relationship between different planes in space and properties of simple solid figures through manipulative activities.
s-III-4	Understand the formula for computing the volume of prisms (including cube and rectangular parallelepiped) and cylinders.
s-III-5	Understand the properties of geometrical figures by way of simple inference.
s-III-6	Recognize the meaning of line symmetry and the inferences based on it.
s-III-7	Recognize the meaning and application of scaling of plane figures.
r-III-1	Understand the operation rules, including the distribution rule, and apply this understanding to compute expressions involving mixed arithmetic operations and solve application problems.
r-III-2	Be proficient in carrying out mixed arithmetic operations of numbers that include fractions and decimals.
r-III-3	Observe the relationships among quantities from models and contexts; and express them using appropriate verbal or symbolic expressions to make inferences and solve problems.
d-III-1	Read off from pie charts, draw line graphs and pie charts, and make simple inferences using these graphical displays of data.
d-III-2	Solve simple problems related to plausibility based either on data or numerical values from graphs.
Stage IV	
n-IV-1	Understand the meaning of factor, multiple, greatest common factor and least common multiple, be competent in their computation, and be able to apply them to solve problems encountered in daily contexts.
n-IV-2	Understand the meaning of negative numbers, their sign and their representation on the number line, be proficient in the four arithmetic operations involving these numbers, as well as to be able to apply them to solve problems encountered in daily contexts.
n-IV-3	Understand exponentials that involve non-negative indices and the exponential rules, be able to apply them to carry out prime factorization and express in scientific notation, and be able to apply them to solve problems encountered in daily contexts.

Code	Learning Performance (arranged according to the Learning Stages)
n-IV-4	Understand the meaning of and inferences based on rate, proportion, direct proportion, indirect proportion and continued proportion, and be able to apply them to solve problems encountered in daily contexts.
n-IV-5	Understand the meaning of square root, its symbol and the four arithmetic operations involving radicals, and be able to apply this knowledge to solve problems encountered in daily contexts.
n-IV-6	Be able to apply the digit-by-digit calculation approach to find approximate values of square roots, to use calculators to verify, compute and estimate an answer, thereby building number sense for square roots.
n-IV-7	Identify the patterns of sequences, be able to use symbols to represent the relationship and pattern among quantities in real life, recognize arithmetic and geometric sequences, as well as to compute the rest of the terms based on the first term and the common difference or ratio.
n-IV-8	Understand the summation formula for arithmetic series, and be able to apply it to solve problems encountered in daily contexts.
n-IV-9	Be able to use calculators to carry out the four arithmetic operations involving ratios, complicated expressions, decimals or radicals etc., to compute approximate values of trigonometric ratios, and be able to understand possible errors that may be produced by calculators.
s-IV-1	Understand the definitions, representations and properties of common geometric forms and figures, and apply this knowledge to solve geometric problems.
s-IV-2	Understand various properties of angles, the meaning of the interior and exterior angles of triangles and polygons, the sum of the exterior angles of triangles, the sum of the interior angles of convex polygons, and be able to apply this knowledge to solve problems encountered in daily life.
s-IV-3	Understand the meaning of perpendicular and parallel relationships between two straight lines as well as their properties, and be able to apply this knowledge to solve geometric problems and problems encountered in daily life.
s-IV-4	Understand the meaning of congruence of plane figures, realize that figures remain congruent after translation, rotation and reflection, and be able to apply this knowledge to solve geometric problems and problems encountered in daily life.
s-IV-5	Understand the meaning of line symmetry and the geometric properties of line symmetric figures, and be able to apply this knowledge to solve geometric problems and other problems encountered in daily life.
s-IV-6	Understand the meaning of similarity of plane figures, realize that a figure is similar to itself after scaling, and be able to apply this knowledge to solve geometric problems and other problems encountered in daily life.
s-IV-7	Understand Pythagoras' theorem and its converse theorem, and be able to apply this knowledge to solve mathematical problems and other problems encountered in daily life.
s-IV-8	Understand the properties and related issues regarding special triangles (for example, equilateral, isosceles and right-angled triangles), special quadrilaterals (for example, square, rectangle, parallelogram, rhombus, kite, trapezium) and regular polygons.
s-IV-9	Understand the relationship between angles and sides of a triangle, be able to use the corresponding equivalence of angles and sides to determine if two triangles are congruent, as well as to apply this knowledge to solve geometric problems and problems encountered in daily life.

Code	Learning Performance (arranged according to the Learning Stages)
s-IV-10	Understand the properties of similarity between two triangles, be able to apply the equivalence of corresponding angles and sides to determine congruence of two triangles, as well as to apply this knowledge to solve geometric problems and also problems encountered in daily life.
s-IV-11	Understand the meaning and properties of the centroid, circumcenter and incenter of a triangle.
s-IV-12	Understand that one of the acute angles of a right-angled triangle can determine the ratio of the sides, recognize the symbols used to represent these ratios, as well as to apply this knowledge to solve problems encountered in daily contexts.
s-IV-13	Understand the narration of operations using straightedge and compass, and be able to apply this knowledge to perform construction with straightedge and compass.
s-IV-14	Recognize the concepts in relation to a circle (for example, radius, chord, arc, segment of a circle) and their geometrical properties (for example, central angle, angle at circumference, opposite angles of an inscribed quadrilateral are supplementary, etc.), as well as to understand the formulae for arc length, area of a circle and area of a sector.
s-IV-15	Recognize the perpendicular and parallel relationships between different straight lines in space and also between a straight line and a plane in space.
s-IV-16	Understand simple solid figures, their three-view drawing and nets, and be able to compute their surface areas, lateral areas and volumes.
g-IV-1	Recognize the meaning of the Cartesian coordinate system and its components, be able to read and mark off coordinate points, as well as to compute the distance between two coordinate points.
g-IV-2	Be able to draw the graph of a linear equation in two unknowns on the Cartesian coordinate plane, and understand the geometric meaning of a unique solution to a simultaneous system of two linear equations in two unknowns.
a-IV-1	Understand and be able to use symbols and verbal narration to communicate concepts, computations, inferences, and proofs.
a-IV-2	Understand the meaning of a linear equation in one unknown and its solution, be able to use the axiom of equality and move terms to solve the equation and verify the solution, as well as to apply this knowledge to solve problems encountered in daily contexts.
a-IV-3	Understand the meaning of a linear inequality in one unknown, be able to identify the range of the solution and mark the corresponding region on a number line, to use the mathematical symbols for inequality to describe a situation, as well as to communicate the results.
a-IV-4	Understand the meaning of two simultaneous linear equations in two unknowns and their solution, be able to use elimination by substitution and elimination by addition or subtraction to solve the equations and verify the solution, and to apply this knowledge to solve problems encountered in daily contexts.
a-IV-5	Recognize polynomial and related terms and be proficient in the four arithmetic operations on polynomials as well as the application of the multiplication formula.
a-IV-6	Understand the meaning of a quadratic equation in one unknown and its solution, be able to use factorization and completing the square to solve and verify the solutions, as well as to apply this knowledge to solve problems encountered in daily contexts.
f-IV-1	Understand the meaning of a constant function and a linear function, be able to draw their corresponding graphs, as well as to apply this knowledge to solve problems encountered in daily contexts.
f-IV-2	Understand the meaning of a quadratic function, and be able to draw its graph on the Cartesian coordinate plane.

Code	Learning Performance (arranged according to the Learning Stages)
f-IV-3	Understand the standard form of the quadratic function and be proficient in solving problems related to whether the curves open upward or downward, the widths of the curves, the vertices, axes of symmetry and optimal values of the quadratic functions.
d-IV-1	Understand common statistical graphs, and be able to use simple statistics to find the characteristics of the data, to use statistical software to compile information representations, as well as to communicate the results.
d-IV-2	Understand the meaning of classical probability, be able to express uncertainty by means of probability and to use the tree diagram to account for all the possible outcomes of an event, and be able to apply them to solve simple problems in daily life contexts.

Stage V	
n-V-1	Understand the relationship between real numbers and the number line, the meaning of decimal representation, the properties of whole numbers, rational numbers and irrational numbers; be proficient in the mixed arithmetic operations on these numbers and the computation of powers; be able to develop a number sense for exponentials and logarithms; be able to use intervals to denote the range on the number line, as well as to use real numbers to describe phenomena and solve problems.
n-V-2	Be proficient in operating a calculator, be able to discern when to use a calculator, understand possible errors produced by using a calculator, as well as to be able to handle such errors.
n-V-3	Recognize complex numbers, understand complex numbers as numbers on a plane, understand that complex numbers are compatible with real numbers except for the trichotomy law, be able to do computations on complex numbers, as well as to use them to describe phenomena and solve problems.
n-V-4	Understand the meaning of taking the absolute value of various kinds of numbers and quantities, and be able to operate on them; be able to appreciate the coherence in meaning from both geometric and algebraic perspectives, and be able to describe phenomena and communicate by means of absolute values.
n-V-5	Be able to observe patterns in a sequence and express them by means of finding the general terms or by using the recursive approach, and subsequently be proficient in handling series. Understand the meaning of mathematical induction and be able to use it to conduct mathematical argumentation.
n-V-6	Recognize propositions, understand and appreciate the consistency and accuracy of logic relative to natural language, and be able to use them to communicate and make inferences.
n-V-7	Recognize radians, and be able to operate on them, as well as understand and appreciate the conciseness in using radian as a measure of angle.
n-V-8	Recognize the concept of infinity, as well as understand and appreciate the mathematical approach to handle infinity.
s-V-1	Understand the meaning of various trigonometric ratios, be proficient in their relationships, and be able to do computations, be flexible in applying this knowledge to identities and functions, and be able to make inferences and solve problems.
s-V-2	Observe and understand the basic properties of space and the relationship among points, straight lines and planes in space, be able to recognize special curves in space, and to identify and appreciate their examples from daily life.
g-V-1	Recognize that in the Cartesian coordinate system, order pairs or triplets of numbers can be used to represent the position of a point in a plane or in space respectively, that operations on points can be done by means of vectors, understand and be proficient in their operations, and be able to apply this knowledge in communication.
g-V-2	Be able to understand and appreciate the symmetry of a figure in the Cartesian plane, and be able to use this knowledge to communicate and make inferences.
g-V-3	Recognize polar coordinates, understand the relationship among the concepts of azimuth, direction and slope, be proficient in transiting between representations of points in Cartesian and polar coordinates, and be able use this knowledge to communicate.
g-V-4	Understand and appreciate that geometric properties can be expressed in terms of quantities and equations by using coordinates. Furthermore, algebraic operations on the quantities and equations based on the coordinates also have corresponding geometric meanings. Be proficient in transiting between geometric and algebraic representations, and be able to use this knowledge to make inferences and solve problems.

g-V-5	Understand and appreciate that the coordinate approach allows some geometric problems to be solved by computation in a concise way. Furthermore, translations and dilation in the coordinate approach can be used to simplify algebraic problems. Be proficient in the abovementioned operations, and be able to use this knowledge to make inferences and solve problems.
a-V-1	Understand the correspondence between the arithmetic operation rules of polynomials, fractional expressions and radicals with respect to those of the real numbers, understand the operation rules of exponentials and logarithms, and be able to use this knowledge to make inferences.
a-V-2	Understand and be proficient in the arithmetic operations on polynomials, be flexible in applying this knowledge to identities or functions, and to use this knowledge to make inferences and solve problems.
a-V-3	Recognize matrices, understand the meaning of linear combinations and matrix operations, and be able to apply this knowledge to solve problems.
a-V-4	Understand the meaning of a range of solutions to an inequality, and be able to apply this understanding to solve problems.
f-V-1	Recognize functions, understand the connection between equations and functions and be flexible with changing from one to the other, understand the meaning of the graph of a function, and be able to communicate using this knowledge.
f-V-2	Recognize the characteristics of the graphs of polynomial functions, understand the meaning of these characteristics, recognize that polynomial functions can be used as mathematical models for certain relationships or phenomena, and be able to use this knowledge to communicate and solve problems.
f-V-3	Recognize the characteristics of the graphs of trigonometric functions, understand the meaning of these characteristics, recognize that sine functions can be used as mathematical models for periodic phenomena, and be able to use this knowledge to communicate and solve problems.
f-V-4	Recognize the characteristics of the graphs of exponential and logarithmic functions, and understand the meaning of these characteristics, recognize that exponential and logarithmic functions can be used as mathematical models for growth and decay phenomena, and be able to use them to communicate and solve problems.
f-V-5	Understand the meaning of applying matrices to represent linear transformations, and be able to use this knowledge to communicate and solve problems.
f-V-6	Recognize the concept of limit, understand the meaning of differentiation and derivative, and be able to use this knowledge to communicate and make inferences.
f-V-7	Understand the meaning of derivative functions, be proficient in their operations, and be able to apply them to solve problems.
f-V-8	Recognize that differentiation and integration are inverse operations of each other, understand the meaning of the Fundamental Theorem of Calculus, and be able to use this knowledge to make inferences.
f-V-9	Understand the principles of definite integral, and be able to use this knowledge to communicate, make inferences and solve problems.
d-V-1	Recognize the concept of set, understand and appreciate the conciseness of the language of set, be able to conduct operations on sets, be able to use Venn diagrams as a supplementary tool, and be able to use this knowledge to communicate and make inferences.
d-V-2	Be able to determine when to conduct data analysis, choose appropriate statistics to describe the parameters behind the data, understand that there are situations when data analyses may produce results that may not be appropriate, and know how to handle these situations.

d-V-3	Understand the uncertainty regarding the outcomes of an event, and be able to quantify it with probability. Understand the properties of probability, be able to carry out computations on probability, and be able to use the knowledge of probability to communicate and make inferences.
d-V-4	Recognize random variables, understand the meaning of their distributions, understand the meaning of parameters and how they are estimated, and be able to use them to make inferences and solve problems.
d-V-5	Be able to use probability to check uncertain hypotheses or the rationality of inferences under uncertainty.
d-V-6	Understand basic counting principles and be able to use strategies and principles to exhaust all possible outcomes of an event.
d-V-7	Recognize the counting models for permutation and combination, understand their principles of operation, and be able to use this knowledge to communicate and solve problems.

Learning performance organized according to topics across different learning stages

Code	Learning performance (organized according to topics)
Number and quantity (n)	
n-I-1	Understand the place value structure for numbers within one thousand, and be able to use it as the foundation for the four fundamental arithmetic operations.
n-I-2	Understand the meaning of addition and subtraction, and be proficient and fluent in their computation.
n-I-3	Apply addition and subtraction through computation and estimation to solve application problems encountered in daily life.
n-I-4	Understand the meaning of multiplication; be proficient in ten-by-ten (10×10) multiplication; and begin to be engaged in activities related to partitive and quotative division.
n-I-5	Solve simple two-step application problems in specific contexts.
n-I-6	Recognize unit fractions.
n-I-7	Understand the concept of length and its common units; perform measurement, estimation and computation.
n-I-8	Recognize the concepts of capacity, weight and area.
n-I-9	Recognize the concepts of moment and duration of time together with their common units.
n-II-1	Understand the place value structure for numbers within a hundred million, and be able to use it as the foundation for various computations and estimations.
n-II-2	Be proficient in adding, subtracting and multiplying relatively larger numbers through computation or estimation, and be able to apply them to solve problems in daily contexts.
n-II-3	Understand the meaning of division, be able to compute and estimate, and be able to apply them to solve problems in daily contexts.
n-II-4	Solve estimation problems involving the four arithmetic operations encountered in daily contexts.
n-II-5	Solve two-step application problems in daily contexts.
n-II-6	Understand the meaning of fractions with the same denominator together with the addition, subtraction and integral multiples of such fractions, and be able to compute and apply them. Recognize the meaning of equivalent fractions and relate it to simple comparisons between fractions with different denominators as well as the meaning of adding and subtracting them.

Code	Learning performance (organized according to topics)
n-II-7	Understand the meaning of decimals and the place value structure, be able to perform addition, subtraction and multiplication in the vertical format, and apply them to solve problems.
n-II-8	Be able to mark off whole numbers, fractions, and decimals on the number line, and compare and perform addition and subtraction. Understand that whole numbers, fractions and decimals are all numbers.
n-II-9	Understand the common units of and conversion involving length, angle, area, capacity and weight.
n-II-10	Understand the principles of addition and subtraction involving time, and be able to apply them to solve problems in daily contexts.
n-III-1	Understand the place value structure of the decimal system, and be able to extend this understanding to very large and very small numbers.
n-III-2	Solve common application problems requiring more than three steps in real life contexts.
n-III-3	Recognize the meanings of factor, multiple, prime number, the greatest common factor and the least common multiple, along with their computation and application.
n-III-4	Understand the meaning of reducing a fraction, expanding a fraction and taking common denominator, and be able to apply these concepts to perform the addition and subtraction of fractions with different denominators.
n-III-5	Understand the meaning of the fractional representation of the division of whole numbers.
n-III-6	Understand the meaning of fraction multiplication and division, also their computation and application.
n-III-7	Understand the meaning of multiplication and division involving decimals, and be able to perform computations in the vertical format and apply them to solve problems.
n-III-8	Understand how to take approximate numbers by rounding and carrying out realistic estimations.
n-III-9	Understand the meaning of a proportional relation, and be able to utilize it to observe, express, compute and solve problems involving rate, proportional scale, speed and baseline amounts.
n-III-10	Explore how to formulate the relationship between quantities in more complex contexts or models into correct mathematical expressions to make inferences or solve problems.
n-III-11	Recognize the common units and the conversion of quantities in relation to area, and be able to solve relevant application problems.
n-III-12	Understand the relationship between capacity and volume, and be able to apply this relationship in problem solving.
n-IV-1	Understand the meaning of factor, multiple, greatest common factor and least common multiple, be competent in their computation, and be able to apply them to solve problems encountered in daily contexts.
n-IV-2	Understand the meaning of negative numbers, their sign and their representation on the number line, be proficient in the four arithmetic operations involving these numbers, as well as to be able to apply them to solve problems encountered in daily contexts.
n-IV-3	Understand exponentials that involve non-negative indices and the exponential rules, be able to apply them to carry out prime factorization and express in scientific notation, and be able to apply them to solve problems encountered in daily contexts.
n-IV-4	Understand the meaning of and inferences based on rate, proportion, direct proportion, indirect proportion and continued proportion, and be able to apply them to solve problems encountered in daily contexts.

Code	Learning performance (organized according to topics)
n-IV-5	Understand the meaning of square root, its symbol and the four arithmetic operations involving radicals, and be able to apply this knowledge to solve problems encountered in daily contexts.
n-IV-6	Be able to apply the digit-by-digit calculation approach to find approximate values of square roots, to use calculators to verify, compute and estimate an answer, thereby building number sense for square roots.
n-IV-7	Identify the patterns of sequences, be able to use symbols to represent the relationship and pattern among quantities in real life, recognize arithmetic and geometric sequences, as well as to compute the rest of the terms based on the first term and the common difference or ratio.
n-IV-8	Understand the summation formula for arithmetic series, and be able to apply it to solve problems encountered in daily contexts.
n-IV-9	Be able to use calculators to carry out the four arithmetic operations involving ratios, complicated expressions, decimals or radicals etc., to compute approximate values of trigonometric ratios, and be able to understand possible errors that may be produced by calculators.
n-V-1	Understand the relationship between real numbers and the number line, the meaning of decimal representation, the properties of whole numbers, rational numbers and irrational numbers; be proficient in the mixed arithmetic operations on these numbers and the computation of powers; be able to develop a number sense for exponentials and logarithms; be able to use intervals to denote the range on the number line, as well as to use real numbers to describe phenomena and solve problems.
n-V-2	Be proficient in operating a calculator, be able to discern when to use a calculator, understand possible errors produced by using a calculator, as well as to be able to handle such errors.
n-V-3	Recognize complex numbers, understand complex numbers as numbers on a plane, understand that complex numbers are compatible with real numbers except for the trichotomy law, be able to do computations on complex numbers, as well as to use them to describe phenomena and solve problems.
n-V-4	Understand the meaning of taking the absolute value of various kinds of numbers and quantities, and be able to operate on them; be able to appreciate the coherence in meaning from both geometric and algebraic perspectives, and be able to describe phenomena and communicate by means of absolute values.
n-V-5	Be able to observe patterns in a sequence and express them by means of finding the general terms or by using the recursive approach, and subsequently be proficient in handling series. Understand the meaning of mathematical induction and be able to use it to conduct mathematical argumentation.
n-V-6	Recognize propositions, understand and appreciate the consistency and accuracy of logic relative to natural language, and be able to use them to communicate and make inferences.
n-V-7	Recognize radians, and be able to operate on them, as well as understand and appreciate the conciseness in using radian as a measure of angle.
n-V-8	Recognize the concept of infinity, as well as understand and appreciate the mathematical approach to handle infinity.
Space and shape (s)	
s-I-1	Attain initial recognition of the geometrical properties of objects and common geometrical forms through manipulative activities.
s-II-1	Understand the formulae of area and perimeter for the square and the rectangle and their application.
s-II-2	Recognize the meaning of congruence of plane figures.

Code	Learning performance (organized according to topics)
s-II-3	Recognize commonly seen triangles, quadrilaterals and circles through their composing constituents.
s-II-4	Recognize the application of geometric concepts such as rotated angles, nets and solid figures through learning activities.
s-III-1	Understand how to compute the area of triangles, parallelograms and trapeziums.
s-III-2	Recognize the meaning of the ratio of the circumference of a circle to its diameter and understand how to compute the area of a circle and its circumference, and the area of a sector and its arc length.
s-III-3	Understand the relationship between different planes in space and properties of simple solid figures through manipulative activities.
s-III-4	Understand the formula for computing the volume of prisms (including cube and rectangular parallelepiped) and cylinders.
s-III-5	Understand the properties of geometrical figures by way of simple inference.
s-III-6	Recognize the meaning of line symmetry and the inferences based on it.
s-III-7	Recognize the meaning and application of scaling of plane figures.
s-IV-1	Understand the definitions, representations and properties of common geometric forms and figures, and apply this knowledge to solve geometric problems.
s-IV-2	Understand various properties of angles, the meaning of the interior and exterior angles of triangles and polygons, the sum of the exterior angles of triangles, the sum of the interior angles of convex polygons, and be able to apply this knowledge to solve problems encountered in daily life.
s-IV-3	Understand the meaning of perpendicular and parallel relationships between two straight lines as well as their properties, and be able to apply this knowledge to solve geometric problems and problems encountered in daily life.
s-IV-4	Understand the meaning of congruence of plane figures, realize that figures remain congruent after translation, rotation and reflection, and be able to apply this knowledge to solve geometric problems and problems encountered in daily life.
s-IV-5	Understand the meaning of line symmetry and the geometric properties of line symmetric figures, and be able to apply this knowledge to solve geometric problems and other problems encountered in daily life.
s-IV-6	Understand the meaning of similarity of plane figures, realize that a figure is similar to itself after scaling, and be able to apply this knowledge to solve geometric problems and other problems encountered in daily life.
s-IV-7	Understand Pythagoras' theorem and its converse theorem, and be able to apply this knowledge to solve mathematical problems and other problems encountered in daily life.
s-IV-8	Understand the properties and related issues regarding special triangles (for example, equilateral, isosceles and right-angled triangles), special quadrilaterals (for example, square, rectangle, parallelogram, rhombus, kite, trapezium) and regular polygons.
s-IV-9	Understand the relationship between angles and sides of a triangle, be able to use the corresponding equivalence of angles and sides to determine if two triangles are congruent, as well as to apply this knowledge to solve geometric problems and problems encountered in daily life.
s-IV-10	Understand the properties of similarity between two triangles, be able to apply the equivalence of corresponding angles and sides to determine congruence of two triangles, as well as to apply this knowledge to solve geometric problems and also problems encountered in daily life.
s-IV-11	Understand the meaning and properties of the centroid, circumcenter and incenter of a triangle.

Code	Learning performance (organized according to topics)
s-IV-12	Understand that one of the acute angles of a right-angled triangle can determine the ratio of the sides, recognize the symbols used to represent these ratios, as well as to apply this knowledge to solve problems encountered in daily contexts.
s-IV-13	Understand the narration of operations using straightedge and compass, and be able to apply this knowledge to perform construction with straightedge and compass.
s-IV-14	Recognize the concepts in relation to a circle (for example, radius, chord, arc, segment of a circle) and their geometrical properties (for example, central angle, angle at circumference, opposite angles of an inscribed quadrilateral are supplementary, etc.), as well as to understand the formulae for arc length, area of a circle and area of a sector.
s-IV-15	Recognize the perpendicular and parallel relationships between different straight lines in space and also between a straight line and a plane in space.
s-IV-16	Understand simple solid figures, their three-view drawing and nets, and be able to compute their surface areas, lateral areas and volumes.
s-V-1	Understand the meaning of various trigonometric ratios, be proficient in their relationships, and be able to do computations, be flexible in applying this knowledge to identities and functions, and be able to make inferences and solve problems.
s-V-2	Observe and understand the basic properties of space and the relationship among points, straight lines and planes in space, be able to recognize special curves in space, and to identify and appreciate their examples from daily life.
Coordinate geometry (g)	
g-IV-1	Recognize the meaning of the Cartesian coordinate system and its components, be able to read and mark off coordinate points, as well as to compute the distance between two coordinate points.
g-IV-2	Be able to draw the graph of a linear equation in two unknowns on the Cartesian coordinate plane, and understand the geometric meaning of a unique solution to a simultaneous system of two linear equations in two unknowns.
g-V-1	Recognize that in the Cartesian coordinate system, order pairs or triplets of numbers can be used to represent the position of a point in a plane or in space respectively, that operations on points can be done by means of vectors, understand and be proficient in their operations, and be able to apply this knowledge in communication.
g-V-2	Be able to understand and appreciate the symmetry of a figure in the Cartesian plane, and be able to use this knowledge to communicate and make inferences.
g-V-3	Recognize polar coordinates, understand the relationship among the concepts of azimuth, direction and slope, be proficient in transiting between representations of points in Cartesian and polar coordinates, and be able use this knowledge to communicate.
g-V-4	Understand and appreciate that geometric properties can be expressed in terms of quantities and equations by using coordinates. Furthermore, algebraic operations on the quantities and equations based on the coordinates also have corresponding geometric meanings. Be proficient in transiting between geometric and algebraic representations, and be able to use this knowledge to make inferences and solve problems.
g-V-5	Understand and appreciate that the coordinate approach allows some geometric problems to be solved by computation in a concise way. Furthermore, translations and dilation in the coordinate approach can be used to simplify algebraic problems. Be proficient in the abovementioned operations, and be able to use this knowledge to make inferences and solve problems.

Code	Learning performance (organized according to topics)
Relation (r)	
r-I-1	Learn symbols of operations, relational symbols and the convention of arithmetic expression as part of the mathematical language.
r-I-2	Recognize the operation rules of addition and multiplication.
r-I-3	Recognize the inverse relationship between addition and subtraction and apply it to solve problems.
r-II-1	Understand the inverse properties of multiplication and division, and be able to apply them and solve problems.
r-II-2	Recognize one-dimensional and two-dimensional quantitative models and be able to explain and make simple inferences.
r-II-3	Understand the convention of combining the parts of a two-step problem into a single expression, and the convention of carrying out computations involving the four arithmetic operations.
r-II-4	Recognize the rules regarding addition and subtraction in a two-step computation problem, the rules regarding multiplication and division (for example, the order of multiplication and division is inconsequential in an expression involving only these operations), and be able to apply these rules to solve problems.
r-II-5	Understand mathematical expressions in verbal format.
r-III-1	Understand the operation rules, including the distribution rule, and apply this understanding to compute expressions involving mixed arithmetic operations and solve application problems.
r-III-2	Be proficient in carrying out mixed arithmetic operations of numbers that include fractions and decimals.
r-III-3	Observe the relationships among quantities from models and contexts; and express them using appropriate verbal or symbolic expressions to make inferences and solve problems.
Algebra (a)	
a-IV-1	Understand and be able to use symbols and verbal narration to communicate concepts, computations, inferences, and proofs.
a-IV-2	Understand the meaning of a linear equation in one unknown and its solution, be able to use the axiom of equality and move terms to solve the equation and verify the solution, as well as to apply this knowledge to solve problems encountered in daily contexts.
a-IV-3	Understand the meaning of a linear inequality in one unknown, be able to identify the range of the solution and mark the corresponding region on a number line, to use the mathematical symbols for inequality to describe a situation, as well as to communicate the results.
a-IV-4	Understand the meaning of two simultaneous linear equations in two unknowns and their solution, be able to use elimination by substitution and elimination by addition or subtraction to solve the equations and verify the solution, and to apply this knowledge to solve problems encountered in daily contexts.
a-IV-5	Recognize polynomial and related terms and be proficient in the four arithmetic operations on polynomials as well as the application of the multiplication formula.
a-IV-6	Understand the meaning of a quadratic equation in one unknown and its solution, be able to use factorization and completing the square to solve and verify the solutions, as well as to apply this knowledge to solve problems encountered in daily contexts.

Code	Learning performance (organized according to topics)
a-V-1	Understand the correspondence between the arithmetic operation rules of polynomials, fractional expressions and radicals with respect to those of the real numbers, understand the operation rules of exponentials and logarithms, and be able to use this knowledge to make inferences.
a-V-2	Understand and be proficient in the arithmetic operations on polynomials, be flexible in applying this knowledge to identities or functions, and to use this knowledge to make inferences and solve problems.
a-V-3	Recognize matrices, understand the meaning of linear combinations and matrix operations, and be able to apply this knowledge to solve problems.
a-V-4	Understand the meaning of a range of solutions to an inequality, and be able to apply this understanding to solve problems.
Function (f)	
f-IV-1	Understand the meaning of a constant function and a linear function, be able to draw their corresponding graphs, as well as to apply this knowledge to solve problems encountered in daily contexts.
f-IV-2	Understand the meaning of a quadratic function, and be able to draw its graph on the Cartesian coordinate plane.
f-IV-3	Understand the standard form of the quadratic function and be proficient in solving problems related to whether the curves open upward or downward, the widths of the curves, the vertices, axes of symmetry and optimal values of the quadratic functions.
f-V-1	Recognize functions, understand the connection between equations and functions and be flexible with changing from one to the other, understand the meaning of the graph of a function, and be able to communicate using this knowledge.
f-V-2	Recognize the characteristics of the graphs of polynomial functions, understand the meaning of these characteristics, recognize that polynomial functions can be used as mathematical models for certain relationships or phenomena, and be able to use this knowledge to communicate and solve problems.
f-V-3	Recognize the characteristics of the graphs of trigonometric functions, understand the meaning of these characteristics, recognize that sine functions can be used as mathematical models for periodic phenomena, and be able to use this knowledge to communicate and solve problems.
f-V-4	Recognize the characteristics of the graphs of exponential and logarithmic functions, and understand the meaning of these characteristics, recognize that exponential and logarithmic functions can be used as mathematical models for growth and decay phenomena, and be able to use them to communicate and solve problems.
f-V-5	Understand the meaning of applying matrices to represent linear transformations, and be able to use this knowledge to communicate and solve problems.
f-V-6	Recognize the concept of limit, understand the meaning of differentiation and derivative, and be able to use this knowledge to communicate and make inferences.
f-V-7	Understand the meaning of derivative functions, be proficient in their operations, and be able to apply them to solve problems.
f-V-8	Recognize that differentiation and integration are inverse operations of each other, understand the meaning of the Fundamental Theorem of Calculus, and be able to use this knowledge to make inferences.
f-V-9	Understand the principles of definite integral, and be able to use this knowledge to communicate, make inferences and solve problems.

Data and uncertainty (d)	
d-I-1	Recognize different modes of classification, able to actively collect, classify and make simple records and explanation.
d-II-1	Read off and produce one- and two-dimension tables and bar-charts, read off from line graphs and make simple inferences.
d-III-1	Read off from pie charts, draw line graphs and pie charts, and make simple inferences using these graphical displays of data.
d-III-2	Solve simple problems related to plausibility based either on data or numerical values from graphs.
d-IV-1	Understand common statistical graphs, and be able to use simple statistics to find the characteristics of the data, to use statistical software to compile information representations, as well as to communicate the results.
d-IV-2	Understand the meaning of classical probability, be able to express uncertainty by means of probability and to use the tree diagram to account for all the possible outcomes of an event, and be able to apply them to solve simple problems in daily life contexts.
d-V-1	Recognize the concept of set, understand and appreciate the conciseness of the language of set, be able to conduct operations on sets, be able to use Venn diagrams as a supplementary tool, and be able to use this knowledge to communicate and make inferences.
d-V-2	Be able to determine when to conduct data analysis, choose appropriate statistics to describe the parameters behind the data, understand that there are situations when data analyses may produce results that may not be appropriate, and know how to handle these situations.
d-V-3	Understand the uncertainty regarding the outcomes of an event, and be able to quantify it with probability. Understand the properties of probability, be able to carry out computations on probability, and be able to use the knowledge of probability to communicate and make inferences.
d-V-4	Recognize random variables, understand the meaning of their distributions, understand the meaning of parameters and how they are estimated, and be able to use them to make inferences and solve problems.
d-V-5	Be able to use probability to check uncertain hypotheses or the rationality of inferences under uncertainty.
d-V-6	Understand basic counting principles and be able to use strategies and principles to exhaust all possible outcomes of an event.
d-V-7	Recognize the counting models for permutation and combination, understand their principles of operation, and be able to use this knowledge to communicate and solve problems.

2. Learning Content

The coding system for the Learning Content is explained as follows:

The first code refers to the main topics of mathematics and is denoted by capital letters, with N representing Number and Quantity, S for Space and Shape, G for Geometry and Coordinate Geometry, R for Relation, A for Algebra, D for Data and Uncertainty. Notice that R is specifically for use in the elementary schools. It is replaced by A and F in the junior and upper

secondary schools.

The second code refers to the grade level and is denoted by Arabic numerals from 1 to 12, each representing the corresponding grade. Notice that for Grade 11, it is separated into two categories, namely, 11A and 11B. As for Grade 12, the broadened and deepened curriculum is separated into the following two categories, 12-I and 12-II.

The third code refers to the serial number. It is not necessary for textbooks to be compiled according to the serial number of the topics within any grade.

Learning Contents are made up of two components, namely, the heading and the description. The former is the name of the item and is denoted in bold, and the latter is an account of the content pertaining to the item. They are separated by a colon. The Learning Contents are organized according to the idea of presenting a group of mathematical concepts in a clear fashion. It should not be so interpreted that each item corresponds to a teaching unit in a strict sense. Some of the entries may require more clarification as specified in the Remarks column. These remarks are as important as the corresponding descriptions, and may even be more direct in pointing out the direction of instruction. When the phrase "problem solving" appears in entries under the "Learning Content Heading and Description" column for Grades 1 to 6, attention should be drawn to two specific points. First, instructors should try to match the spirit of problem solving by referring to the entry code listed in the column "Corresponding Learning Performance." Second, instructors should pay attention to connecting mathematics learning with applications to daily life. For any grade, if such phrases as "but not beyond" or "not required" appear in the "Remarks" column, then the specified content in relation to these phrases should not be covered at that grade so as not to disrupt the curricular arrangement for subsequent grades.

For topics that were taught in the previous curricula for Grades 7 to 12 but deleted in the present curriculum, these topics are considered harder ones and should not be covered in textbooks. If necessary, they can be delineated in the teachers' manuals that accompany the textbooks. These materials are provided as reference materials to instructors so that they can cater their instruction for advanced students. However, instructors should pay attention to the time management as well as provide proper learning contexts to introduce these materials.

Appropriate use of teaching aids in classrooms can help students understand mathematics visually and conceptually, thereby enhancing the effectiveness of teaching. In order to promote their usage, a list of teaching devices is listed in the column "Teaching Aids" whenever

appropriate. Any teaching device that provides the same instructional function and meet the same teaching goal can be used. It is preferred that teaching aids are either self-made or purchased by students. Simple teaching devices can be designed by the instructors and students together using materials at hand in a flexible manner, whereas more complicated teaching aids should be provided by schools or by competent authorities.

The last column of the table lists the learning performances expected of students for ready reference.

Notice that for some entries in relation to Grades 7-12, there are three special symbols (※, ★ and ＃) that appear under the two columns "Learning Contents Heading and Description" and "Remarks. " Their meanings are as follows:

The symbol ※ denotes advanced or extended materials. Instructors are advised to provide enough explanations and supplementary information when covering them. These materials will not be included in public examinations but can be included in classroom assessment.

The symbol ★ denotes materials that are suggested not to be directly assessed, but can appear in the assessment of other topics when appropriate.

The symbol ＃ denotes materials that are suggested not to be covered as an independent unit, but can be merged within other appropriate topics.

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
Grade 1				
N-1-1	**Numbers within a hundred:** Instruction should include manipulative activities. Use numbers to represent quantity and order. Integrate the concepts of counting, place representation and place value table. Introduce place value (unit) in terms of "ones" and "tens", together with the exchange of place value (units). Recognize the meaning of 0 as a place value.	Instruction can focus on counting numbers up to a hundred but not beyond. Instructional activities can include counting blocks that represent ones or tens in order to teach the concept of place value, and the meaning of 0 as a place value that can facilitate students' future learning of the vertical format of addition.	place value table, base ten blocks, flower counter	n-I-1
N-1-2	**Addition and subtraction:** Introduce the meanings and applications of addition and	Instruction should emphasize on the learning of the combine type	flower counters	n-I-2

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	subtraction, including adding-on, combining, taking away and compare types of application problems. Introduce the addition and subtraction expressions.	(composite type) of addition so as to help students understand the commutative rule of addition. Instruction should include a treatment on the addition and subtraction involving 0, and also on how to deal with problems that involve both addition and subtraction. It is not necessary in the first grade to treat problems in which the augend, addend, subtractor or minuend is unknown (N-2-3).		
N-1-3	**Basic addition and subtraction:** Mainly through manipulative activities, referring to the addition and reverse subtraction of numbers within 1 to 10. The goal is for students to be proficient in these aspects.	Within instructional activities, it is common to encounter mixed operations of addition and subtraction that involve more than two steps. Computational strategies that extends naturally and the development of number sense are to be encouraged. These kinds of activities are not the formal instruction of two steps computation.	Cards that add up to ten (playing cards)	n-I-2
N-1-4	**Problem solving: Solve problems involving 1, 5, 10, 50 and 100 dollars.** Mainly through manipulative activities, such as counting, exchanging and changing money.	Allow multiple heuristics in order to build up number sense of students. Instructors should not regard this entry as straightforward addition and subtraction units.	Coins	n-I-3
N-1-5	**Length (same as S-1-1):** Mainly through manipulative activities, involving first acquaintance, direct comparison and indirect comparison (including unconventional measuring units)	Activities should include both straight lines and curves. This entry does not require focusing on commonly used measuring units (N-2-11).	strings	n-I-7

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
N-1-6	**Common daily terms about time:** Mainly through manipulative activities. Read off basic date information in relation to month-and-day; tomorrow, today, and yesterday; morning, noon, afternoon, and evening. Tell time in terms of whole-hour and half-hour.	As a guideline, activities should concern with the terminologies of time in classroom conversation between teachers and students. The instruction here does not touch on the structure of time unit (N-2-13、N-2-14). Basic date information refers to the month-and-date from the calendar and does not involve the calendar system. The moment of time should be taught by way of the clock surface and confined only to whole-hour and half-hour.	Calendar and monthly calendar, clock (hour-hand, minute-hand)	n-I-9
S-1-1	◦ **Length (same as N-1-5):** Mainly through manipulative activities, involving first acquaintance, direct comparison and indirect comparison (including unconventional measuring units)	Same as remarks to N-1-5.		n-I-7
S-1-2	**Operations on forms:** Mainly through manipulative activities that involved drawing, duplicating, collaging, and piling up.	Should include activities of plane figures, solid figures, and both, thereby enriching students' geometrical experiences.	different kinds of plane figures , solid figures, puzzles	s-I-1
R-1-1	**Expression and symbols:** Instruction should include the numbers, plus sign, minus sign and the equal sign in addition and subtraction expressions. Assessment of students' understanding can be conducted via speaking, reading, listening and writing activities. This way of assessment is also suitable for the later stages.	This entry extends to learning in subsequent primary stages and will not be listed repeatedly. This entry should be implemented within the units on addition and subtraction and hence not necessary to set up a separate instructional unit solely for this purpose.		r-I-1
R-1-2	**Commutative rule of addition:** The order of	Start with the combine type (composite type) of problem		r-I-2

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	addition between two numbers does not affect their sum. This entry can be immersed within other instructional activities.	contexts and then apply to other contexts. Do not present the formal terminology of "commutative rule of addition".		
D-1-1	**Simple classification:** Mainly use manipulative activities to collect, classify, record and present daily items, as well as read off and describe categories that have already been classified. Observe the different modes of classifications and know that there are different ways to classify the same set of objects.	The diagrams that appear in the activities of this entry are intended to be created works by students and not formal tables (D-3-1) nor statistical diagrams (D-4-1 and the subsequent entries).		d-I-1
Grade 2				
N-2-1	**Numbers within a thousand:** Including manipulative activities on place value blocks. Integrate the concepts of counting, place value representation, place value table. Introduce place value (unit) in terms of "hundreds, " together with the conversion of place value (units).	Instruction can involve counting up to a thousand, but not beyond. Students should be able to count forward or backward from a certain number, count base 10 blocks, and be proficient in counting by tens and by hundreds.	place value table, base 10 blocks	n-I-1
N-2-2	**Addition and subtraction expressions and vertical format of computation:** Use the concept of place value to understand the principle and methods of multidigit addition and subtraction. At the beginning, the manipulative, vertical and horizontal approaches of computation can be presented side by side. The focus of the second grade	Regardless of horizontal or vertical format, addition problems should involve carrying digits twice and subtraction problems only once. Problems should include cases with the digit 0 in numbers. In order to get familiarize with the relationship between place values and vertical format of computation, students should start by learning how to record and compute		n-I-2

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	is eventually on the vertical format as a basis to carry out computation of numbers with more digits. The basis of the vertical format of computation is built on the concept of place value and basic rules of addition and subtraction. Instructors should explain the rationality behind the vertical format of computation.	in place value table.		
N-2-3	**Problem solving: Solve problems involving addition and subtraction.** Application problems with the augend, addend, subtractor or minuend being unknown. Instructors should help students connect the addition operation with the subtraction operation (R-2-4)	Instructors should help students to use heuristics to understand and transform problems (via flower counter model, bar diagram, expressions with unknowns presented as empty squares, or conversion between addition and subtraction). These heuristics need not be developed into fixed format to solve problems. This entry does not require a separate unit for instruction.		n-I-3
N-2-4	**Problem solving: Simple estimation problems in relation to addition and subtraction.** Problems posed should be related to real-life contexts. Focus is on estimation that involves the hundred place.	Problem posing for estimation should relate closely to real-life contexts.		n-I-3
N-2-5	**Problem solving: 100 dollars, 500 dollars and 1000 dollars.** Instruction should be done mainly through manipulative activities that also take into account of computation, allowing for multiple strategies and building up of number sense. Contents should also include smaller	This entry can be implemented in relation to estimation (N-2-4).	coins	n-I-3

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	amounts of money that have already been learned.			
N-2-6	**Multiplication: The meaning and application of multiplication.** While teaching multiplication, instructors should gradually develop the concept of "times, " and then use it as the overarching terminology across different contexts of applications.	When solving problems within a multiplicative context, one can naturally resort to repeated addition without any constraint on the number of steps. Students should eventually be able to use the array model to understand the commutative rule of multiplication.	flower counters (lattice diagrams)	n-I-4
N-2-7	**Ten-by-ten multiplication:** This is the foundation to the vertical format of multiplication and division. The goal is to help students to be proficient in this topic.	This entry should be learned simultaneously with the concept of multiplication. Do not mandate students to commit the multiplication table to rote memory. The learning of this entry should enable students to realize the relationship between multiplication and division within the context of division (refer to N-2-9).		n-I-4
N-2-8	**Problem solving: Two-step application problems (on addition, subtraction and multiplication)**. Instruction should include mixed problems on addition and subtraction, addition and multiplication, subtraction and multiplication, yet excluding any content related to repeated multiplication and the combination of multiple-step operations into a single expression.	Repeated multiplication will be covered in the third grade (N-3-7).		n-I-5
N-2-9	**Problem solving: Chunking and equal sharing.** Mainly through manipulative activities. These activities will constitute precursory experiences for students to	This entry is not concerned about direct teaching of division. No division expression is supposed to be listed and no terminology of division to be mentioned (N-3-4). Coverage should be	flower counters	n-I-4

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	learn division. Students should understand the meaning and methods of chunking and fair sharing. They should be guided, during the problem solving process, to discover that this kind of problems are related to the multiplication model.	confined to situations that allow for wholly divisibility. Instruction should be conducted within the topic of ten-by-ten multiplication, assisted by the strategies of "counting by a number" or by repeated subtraction. These strategies are not to be taken as fixed approaches to solutions.		
N-2-10	**Familiarity with unit fraction:** Instruction should help students to recognize the idea that a part as compared to the whole is regarded as "one out of how many equal parts" based on activities in equal distribution (such as paper folding). Students should know the meanings of such common expressions as "half of, " "a half" and "a quarter" that are used in daily communication. They should be able to explain in an equally dissected figure, one part is "one out of how many parts" with respect to the whole figure.	Students should understand the purpose of equal distribution activities. The activities and instruction on fractions in second grade are confined to continuous quantity. Discrete quantity should be skipped to avoid confusion with N-2-9. Paper folding activities should be confined to operations that fold in half. For example, it is appropriate to use rectangular paper to fold out unit fractions with 2, 4 and 8 as the denominators, and using circular paper to fold out unit fractions with 2 and 4 as the denominators. In accordance to the spirit behind the activities on equal dissection, instructors should mainly use rectangular or circular papers when they want to use equally dissected (grid) diagrams.	Circular and rectangular shapes for paper folding, fraction graphics that are already dissected in circular and rectangular shapes.	n-I-6
N-2-11	**Length: Units in centimeter and meter.** Activities should include direct measurement, quantity sense, measurement estimation, computation, and the conversion of units.	According to the restriction laid out in N-2-1, the number of meters are confined to single digit in problems on changing units. Problems on addition and subtraction of length should include those that	Ruler, set triangle, tape ruler (flexible objects)	n-I-7

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
		can be associated with the number line (N-3-11.)		
N-2-12	**Capacity, Volume and Area:** Mainly through manipulative activities. The instruction on quantity at this stage should include initial recognition, direct comparison, and indirect comparison (including unconventional measuring units). Different quantities should be treated in separate learning units.	Although the instruction of weight is chiefly by way of the beam balance, students should have direct experiences of weight in a quantity sense. Instruction of this entry does not need to focus on common units (N-3-14、N-3-15、N-3-16).	Containers (including those with equal capacity but different shapes), beam balance and weights, objects of same size but different weights, hundred grid daigram.	n-I-8
N-2-13	**Moments on clock surfaces:** Mainly through manipulative activities, using the positions of the hour-hand and minute-hand on the clock surface to tell the time according to the hour and the minute. Content also includes counting the number of elapsed hours between two different whole hours, which serves as precursory experiences to the subsequent learning of addition and subtraction of time.	Concurrent emphasis on the concepts of "take five for each count" and "take ten for each count". This entry does not include the teaching of the second-hand. The teaching of the concept of "o'clock" should be by way of the clock surface.	Clock surfaces teaching aids	n-I-9
N-2-14	**Time: Year, month, week and day.** Students should understand the relationship and the convention in relation to different units of time.	This entry can include simple computational problem (eg. total number of days for the summer vacation), but it is not necessary to handle general problems on time interval. Coverage can include guiding students to observe the structure of monthly calendar, and teaching the concept of leap year by confining only to the case of one leap year per every four years.	monthly calendar, calendar	n-I-9

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
S-2-1	**Geometrical features of objects:** Mainly through manipulative activities, including identification and description. Instruction should help students to recognize simple geometrical shapes through real objects (including plane figures and solid shapes), and connect with common geometrical concepts such as long, short, big, small, etc.	The activities for this entry should mainly involve real objects. Here, geometric features refer to non-rigorously defined concepts of vertex, angle, edge, plane, perimeter, the interior and the exterior of geometrical figures.		s-I-1
S-2-2	**Simple geometrical shapes:** Mainly through manipulative activities, including the recognition, description and classification of plane figures and solid shapes according to their geometrical features.	Instruction can include the naming of figures and shapes for the purpose of communication. However, these activities should not be developed into lessons about the rigorous definitions of figures and shapes (S-4-7, S-4-8). This entry can fit into the instruction on the classification and presentation of data (D-2-1).	Various simple geometric figures	s-I-1
S-2-3	**Operation with ruler:** Instruction should include measuring length, reading off number of centimeters, and drawing line segment of given length.	Instruction should help students to build up the good habits in learning and using measurement tools starting with this entry. All measurements have errors. Instructors should pay attention to discern the difference between making measurement errors and being wrong in instructional activities as well as in assessment.	ruler	n-I-7
S-2-4	**Side lengths of plane figures:** Mainly through manipulative activities and direct measurement with rulers. Recognize the relationship of side lengths of specific geometrical figures as well as	The computation of perimeter involves a straightforward sequence of addition and is not confined to the restriction of two steps addition. This entry concerns with manipulation and simple computation.		n-I-7

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	computation of their perimeters.	No formula are required here.		
S-2-5	**Area:** Mainly through concrete operations for preliminary recognition, direct comparison, indirect comparison (including the use of unconventional measuring units)	This entry coincides with part of the content of N-2-12.		n-I-8
R-2-1	**Size relationship and the transitive rule:** Instruction should help students to understand the meaning of the comparison symbols "＞" and "＜" in expressions, and the transitive relation of size comparison.	The term "transitive rule" should not appear in instructional activities. This entry should be implemented within the units on addition and subtraction, and not as a separate unit.		r-I-1
R-2-2	**The sum of three numbers is irrelevant with respect to the order of the addition:** Instruction should include the combination of commutative and associative rules of addition. This content can be immersed within other instructional activities.	This entry is explained first through the "combine type" (or "composite type") of problem situations. The term "associative rule" should not appear explicitly in instructional activities.		r-I-2
R-2-3	**The product of two numbers is irrelevant with respect to the order of multiplication: Commutative rule of multiplication.** The content of this entry can be immersed within other instructional activities.	The commutative rule of multiplication should not be taught too early. It is suggested to be taught towards the end of the second grade using the array model. The term "commutative rule of multiplication" should not appear explicitly in instructional activities.		r-I-2
R-2-4	**Relationship between addition and subtraction:** Introduce the inverse relationship between addition and subtraction and apply this relationship to solve problems and verify the answers.	When applying the inversion relationship between addition and subtraction for the purpose of verification, it is only required to use addition to verify the answer from subtraction and not vice versa.		n-I-3 r-I-3

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
D-2-1	**Classification and presentation:** Mainly through manipulative activities that teach students to collect, classify, record and present information in relation to daily objects or geometrical shapes. Instruction should include a discussion about the possibilities of sub-classifications after a previous round of classification.	Coverage should include instruction on informal tables and statistical tables (see the remarks of D-1-1). This entry can fit into the instructional activities for plane figures and solid shapes (S-2-2).	simple plane figures and solid figures (same color)	d-I-1
Grade 3				
N-3-1	**Numbers within ten thousand:** Instruction should include manipulative activities on place value blocks, together with counting, place value representation and place value table. Introduce place value in terms of "thousands", together with the exchange of place value.	Instruction can cover numbers up to ten thousand but not beyond.	place value table	n-II-1
N-3-2	**Addition and subtraction in vertical forma**t: Instruction should include activities with multiple occasions for carrying over and borrowing from the next column in addition and subtraction.	Coverage should include how to deal with numbers having 0 as one of the digits in addition and subtraction. Instruction can start by doing computation in tables with place values.		n-II-2
N-3-3	**Multiply by single digit:** Vertical format of multiplication. Instructors should explain the rationality behind the vertical format of computation. The multiplicand should be double or triple digits.	Coverage should handle problems with the multiplicand having 0 as one of its digit. Instruction can start by doing computation in place value tables. Eventually, students should be able to directly compute the answer in one row. The understanding of implicit distributive rule should come from the experiences of manipulative activities and number sense, and not		n-II-2

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
		from explicit instruction on the distributive rule.		
N-3-4	**Division:** The meaning and the application of division. Based on what is learned under N-2-9, students should comprehend how to use multiplication to solve division problems by using the strategy of taking away a certain number for each count. Students should be proficient to perform division within the range of ten-by-ten multiplication, which is the basis to estimate the quotient.	It is suggested that instructors should treat the situation of wholly division first and then the situation of division with remainder. Instruction should include units that require the application of multiplication and division at the same time, thereby allowing students to actively discern the differences between the two kinds of problems.	flower counter	n-II-3
N-3-5	**Division by a single digit number:** Division by using the vertical format. Instructors should explain the rationality behind the vertical format of computation through the concept of place value. The dividend should be numbers with either double or triple digits.	Instruction should the treatment of problems with the dividend having 0 as one of its digit.		n-II-3
N-3-6	**Problem solving: Application problems by multiplication and division**. Instruction should include solving application problems involving the multiplier, multiplicand, divisor or dividend being unknown. Instruction should help students understand the relationship between multiplication and division (R-3-1).	Instructors can use problem solving strategies to help students understand and transform problems (e.g. the terminology involving "how many times", expressions involving empty blanks, the inversion between multiplication and division, etc.). This entry does not require setting up a separate teaching unit.		n-II-2 n-II-3
N-3-7	**Problem solving: Two steps application problems (addition/subtraction and division, repeated multiplication)**. Instruction should include solving problems that involve repeated multiplication,	Problems that require repeated divisions, and problems that require both multiplication and division are postponed to the fourth grade (N-4-3).		n-II-5

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	addition and division, subtraction and division, but excluding problems that combine any two operations into a single expression.			
N-3-8	**Problem solving: Estimation involving the four operations.** Problems solving in the real-life context. Introduce strategies that can estimate numbers with multiple digits and how to use estimation to verify the reasonableness of answers from the computation.	The context of problem posing in relation to estimation should be close to students' life experiences. Since students begin to learn division at the third grade, estimation problems should be very straightforward at this stage.		n-II-4
N-3-9	**Simple fractions with a common denominator:** By using manipulative activities and appealing to students' former experiences with whole number, instruction should introduce the meaning of comparing, adding and subtracting simple fractions with a common denominator. At this stage, the fractions involved and also the results from operations should be no bigger than two in numerical value. Activities should help students understand that using a unit fraction as the counting basis is analogous to using 1 as the unit to count whole numbers, and that carrying out comparison, addition and subtraction of unit fractions is similar to the case for whole numbers. Students should understand what it means to have the fractions sum up to one.	Fractions are introduced in the third grade without explicitly mentioning the terms proper and improper fractions, and their discussion should be done separately (for example, with the proper fraction in the first semester and the improper fraction in the second.) Mixed fractions should not be discussed at this stage (N-4-5). The initial applications of fractions should use examples that involve continuous quantities. Should the situation call for the handling of contexts involving discrete quantities, instruction should only consider those contexts with their representations very close to being continuous, and the total number of counting units should be used as the denominators of the unit fractions. (For example when there are six pieces of crackers in one box, then only fractions	Fractional pie charts	n-II-6

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
		having 6 as the denominator will be considered, ignoring those situations that involve 2 or 3 as the denominators).		
N-3-10	**Numbers with one decimal:** Recognize the notion of decimal and the symbol for the decimal point through the use of counting, place value representation and place value table. Only the tenth unit is discussed via activities that involve comparing, adding and subtracting (including the vertical format) and problem solving.	The learning of decimals should be closely tied with students' former experiences with whole numbers. The context for application of decimal should basically involve continuous quantities.	place value table	n-II-7
N-3-11	**Number line of whole number**s: Recognize the number line, including reading whole numbers and marking whole numbers on it. It is used to relate with the experiences of order, length and scale. Students should understand the meaning of making comparison as well as adding and subtracting on the number line.	Number line should start from 0. Students should understand the meaning of adding and subtracting on the number line by applying the concept of adding and subtracting lengths (N-2-11).	Number lines teaching aids	n-II-8
N-3-12	**Length: New unit in millimeter.** Activities should include direct measurement, quantity sense, measurement estimation, computation and the conversion of units.	Due to the constraint of N-3-1, the number of meters is restricted to single digit in accordance to the specification of N-3-1. Staring from grade three, compound number is allowed in the addition and subtraction of quantities, but multiplication and division are not allowed.	One meter ruler (with millimeter markings)	n-II-9
N-3-13	**Angle and degree (Same as S-3-1):** Mainly through manipulative activities to help students attain initial recognition of angles and degrees. Activities should	Students can recognize and duplicate a right angle by using a ruler or the right angle of a set square. The common misconceptions in confusing the length of the		n-II-9

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	include direct and indirect comparison of angles and the recognition of right angle.	sides or the "areas" of the angles as reflecting the sizes of given angles should be dealt with during instruction.		
N-3-14	**Area: Unit in square centimeter.** Activities should include direct measurement, quantity sense, measurement estimation and computation.	Students should be able to apply the square centimeter board (10×10 grid diagram) to help enumerate the area of simple figures such as a square, a rectangle and a triangle. It is not necessary to develop a general formula for this entry.	10×10 grid diagram (each grid of 1 square centimeter)	n-II-9
N-3-15	**Capacity: Units in liter and milliliter.** Activities should include direct measurement, quantity sense, measurement estimation, computation and the conversion of units.	Due to the restriction specified in N-3-1, unit conversion into milliliters are confined to capacities that are less than 10 liters. Compound numbers can be used to facilitate the addition and subtraction of capacities. (It is not necessary, however, to perform multiplication and division on compound numbers.)	3-liter and 1-liter measuring cups	n-II-9
N-3-16	**Weight: Units in kilogram and gram.** Activities should include direct measurement, quantity sense, measurement estimation, computation and the conversion of units.	Due to the restriction specified in N-3-1, unit conversion into grams are confined to weight that are less than 10 kilograms. Compound numbers can be used to facilitate the addition and subtraction of weight. (It is not necessary, however, to perform multiplication and division on compound numbers.)	3-kilogram and 1 kilogram scales	n-II-9
N-3-17	**Time: Units in day, hour, minute and second.** Activities should include direct measurement, quantity sense, measurement estimation, computation and the conversion of units. Students should recognize various types of problems	Basically, this entry is concerned with helping students to recognize various types of problems involving the addition and subtraction of time (refer to N-4-13 for deeper issues). Instruction should include treatment of common problems in relation to the	Clock (hour-hand, minute-hand, second-hand)	n-II-10

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	in relation to the addition and subtraction of time.	addition and subtraction of moments in time and the amounts of time. For third grade, addition and subtraction is limited to: first, the amount of times having the same units; and second, compound number can only involve hours and minutes (without carrying over to or borrowing from the other unit).		
S-3-1	**Angle and degree (same as N-3-13):** Mainly through manipulative activities to help students attain initial recognition of angles and degrees. Activities should include direct and indirect comparison of angles and the recognition of right angle.	Same as the remarks to N-3-13.		n-II-9
S-3-2	**Square and rectangle:** Instruction should enable students to use the characteristics of sides and angles to define a square and a rectangle.	Students should know how to determine whether it is the same figure if a square or a rectangle is oriented in a slanting position.		s-II-1
S-3-3	**Circle:** Introduction to the following terms: center, circumference, radius and diameter. Students should be able to use a compass to draw a circle with a given radius.	Students should know that the center is an essential element in the definition of a circle, yet it is not a part of the circle.		s-II-3
S-3-4	**Manipulation on geometrical shapes:** Mainly through manipulative activities. Instruction should include the dissection and recombination of plane figures, and providing students with initial experiences in pasting nets into solid figures. Students	The purpose of this entry is to provide students operational experiences on the relationship between plane figures and to develop their spatial sense. It is intended to inspire students' interest in inquiry and making discovery, and not on formal induction of results in a mathematical	Various nets	s-II-4

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	should know that different shapes of nets may be able to paste into solid figures of the same shape.	sense. Activities on nets are intended for initial experiences, hence it is not necessary to include activities that require complicated reasoning. This entry does not require assessment beyond operational activities.		
R-3-1	**The relationship between multiplication and division:** Students should understand the inverse relationship between multiplication and division and utilize it to solve problems and check answers.	Students can understand that questions like "what multiple of 3 makes 15" and "four times which number will make 12" have to be solved by writing division expressions.		r-II-1
R-3-2	**Number pattern and reasoning (I):** Mainly through manipulative activities that involve observation and reasoning of changes in one dimensional patterns, such as number sequences and one dimensional tables.	This entry only concerns simple reasoning and explanation provided by students. No formula need to be presented in instructional activities, as it is not the purpose of this entry. It can be implemented in association with the instruction on one dimensional and two dimensional tables (D-3-1).		r-II-2
D-3-1	**One dimensional table and two dimensional table:** Mainly through manipulative activities, including the reading, describing and creating tables in everyday life. Two dimensional table will include cross tabulation table.	Table construction is not limited to statistical tables of data from everyday life, but also for tables that allow the observation of changes in number patterns (R-3-2).		d-II-1
Grade 4				
N-4-1	**Numbers up to a hundred million:** Place value units in ten thousand, hundred thousand, million and ten million. Instruction should include helping students to	Instruction can involve numbers up to a hundred million but not beyond.	Place value table	n-II-1

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	build up the habit of adding large numbers, for example, the addition and subtraction between "thirty million one thousand and two hundred" with "twenty-one million and three hundred. " **Note:** The scale of numbers commonly used in Chinese language are somewhat different from those in English.			
N-4-2	**Multiplication and division of larger numbers:** This entry involves the treatment of computation when the multiplier and the divisors have multiple digits in a vertical format. Instructors should explain the rationality behind the computation in vertical format by means of the concept of place value.	Instruction is confined to the multiplication and division of three-digit numbers in the vertical format, with due attention to the role of 0 as a place value. Estimation skills should also be introduced when teaching students to do division involving larger numbers. Students should know that expressions such as 1600×200 and 60000÷400 can be computed in an easier way.		n-II-2 n-II-3
N-4-3	**Problem solving: Two-step application problems.** Instruction on solving application problems that involve both multiplication and division, and also two divisions in a sequence.	Since there are two kinds of division, namely, partitive and quotitive division, instruction should pay attention to the diversity of item types. This entry can be implemented together with the instruction on combining a two-step problem into a single expression (R-4-1).		n-II-5 r-II-3
N-4-4	**Problem solving: Taking approximates from large numbers.** This is to be taught using examples from the everyday life context through the processes of rounding off, rounding up and rounding down. Coverage should include estimating an answer through making	Problems to be solved by estimation should be posed within the everyday life context. This entry should include taking approximates while estimating an answer in problems involving the four operations.		n-II-4

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	approximation in computation and also applying the symbol ≈ for approximation.			
N-4-5	**Fractions with the same denominator:** Instruction on fractions with the same denominator together with an introduction to the following terms: proper fraction, improper fraction and mixed fraction. Coverage will include the conversion among improper fractions and mixed fractions, and comparisons among fractions with the same denominator as well as the addition, subtraction and integral multiples of such fractions.	The values of both the numerators and denominators for this entry need not be large. The purpose is to help students to be fluent in terms of calculating among fractions with the same denominator. It is suggested not to impose on students to obtain an integral multiple of a mixed fraction by first changing the mixed fraction into an improper fraction. Rather, it is suggested, in the spirit of number sense and former experiences from manipulative operations, to help students to do this implicitly by taking multiple of the whole number portion and the fraction portion. This entry, however, is not intended to be a formal instruction on the distributive rule of multiplication.	Fractional pie chart	n-II-6
N-4-6	**Equivalent fraction:** Students should understand the meaning of equivalent fractions through manipulative activities. The coverage should include simple comparison of fractions with different denominators as well as the meaning behind adding and subtracting them. This entry will also include simple conversion from fractions to decimals and vice versa.	Simple fractions with different denominators under this entry refer to the case when one denominator is a multiple of the other denominator. Simple conversion from fractions to decimals refer to the cases when the denominators are equal to either 2, 5, 10 and 100.	Fractional pie chart	n-II-6
N-4-7	**Numbers with two decimals:** Decimal place value in the hundredth.	The learning of decimals should connect closely to students' former	Place value table	n-II-7

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	Learning activities should include the conversion among place values, together with the comparison, computation and problem solving that involve numbers with two decimal places. Furthermore, the addition, subtraction and integral multiple of numbers with two decimal places should be introduced using the vertical format.	experiences with whole numbers. Attention should be directed towards the position of the decimal points while performing computation in the vertical format. The context for application problems of decimals should refer basically to the case of continuous quantities.		
N-4-8	**Number line and fraction and decimal:** Instruction should help students to connect their former experiences of fractions, decimals and lengths. Instruction should help students to realize that whole numbers, fractions and decimals all belong to the same system of numbers through such activities as labeling on the number line as well as simple comparison and computation	Label numbers with only one decimal place (equivalent to having 10 as the denominator) and fractions with the denominators not bigger than 5 on the number line. Thinking along the line of equivalent fractions (N-4-6), instruction should help students to realize that whole numbers, fractions and decimals all comprise the same system. Since students just learn about equivalent fractions in the fourth grade, this entry does not require mixed computation involving both fractions and decimals.	Number line	n-II-8
N-4-9	**Length: Unit in kilometer.** Learning activities should take into account of applications in daily life, including the conversion between various units of length and their computation.	Instruction should consistently help students to build up their quantity sense, including the case of handling larger units of measurement (length).		n-II-9
N-4-10	**Angle: Unit in degree (same as N-4-10).** Instruction should include the operation of a protractor, together with the measurement, estimation and computation of angles.	Instruction on the operation of protractors should include activities that involve the left and right sides of the protractors to measure angles.	Protractor	n-II-9

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	Students should recognize angles between 180 to 360 degrees through the combination of a number of other angles. They should know what are straight angle and round angle, as well as how to construct angles of given degrees.			
N-4-11	**Area: Unit in square meter.** Activities should involve measurement, building quantity sense, measurement estimation and computation involving area.	In accordance to N-4-2, this entry excludes problems that involve the conversion from square centimeter to square meter. No computation involving compound numbers are required.	Square meter board (100 x 100 grid board)	n-II-9
N-4-12	**Volume and cubic centimeter:** Mainly through manipulative activities. Instruction should help students understand the concept of volume through the combination of unit cubes each of one cubic centimeter.	Instructors should be aware that it is not easy to directly or indirectly compare the volumes of two objects. Instruction of volume should go hand-in-hand with the combination of unit cubes each of one cubic centimeter.	Cube	n-II-9
N-4-13	**Problem solving: Addition and subtraction of time in everyday life:** Coverage includes the computation of time that involves two moments in two different hours, before and after noon, in two different days and in terms of the 24-hour clock, together with the conversion of time units.	Instruction should include various types of problems that involves the addition and subtraction of time. It is suggested that instructors may want to avoid tackling the problem of time lag by taking care of it through proper problem posing.	Electronic clock, digital clock on computer screen	n-II-10
S-4-1	**Angle: In degree (same as N-4-10).** Instruction should include the operation of a protractor, together with the measurement, estimation and computation of angles. Students should recognize angles between 180 to 360 degrees through the combination of a number of other angles. They should know what are straight angle and round angle, as	Same as the remarks to N-4-10.	Protractor	n-II-9

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	well as how to construct angles of given degrees.			
S-4-2	**Problem solving: Angle of rotation**. Mainly through manipulative activities and also through computation. Instruction can use a traditional clock as a model to discuss the angle required to bring the initial side to coincide with the terminal side by rotation. Discussion should include the two directions of rotation (clockwise and anticlockwise), and the concepts of straight angle and round angle.	There is no need to deal with problems involving angles greater than 360 degrees.	Clock surface, protractor	s-II-4
S-4-3	**Area and perimeter of squares and rectangles:** Students should understand the relationship between a side length of a square or rectangle with its perimeter or area. They should be able to understand the formula as well as how to apply them, including on simple composite figures.	Use only whole number as side lengths. Students should eventually be able to compute the areas using multiplication according to the definitions, and not by measurement. Simple composite figures are confined to the combination of two geometric figures.		s-II-1
S-4-4	**Volume:** Mainly through manipulative activities that enable students to recognize the meaning of volume and compare them. Coverage should include the recognition of unit cubes of 1 cubic centimeter and an understanding about how to find volumes by enumerating the number of unit cubes.	Same as the remarks to N-4-12.	Cube	n-II-9
S-4-5	**Perpendicular and parallel:** Mainly through manipulative activities. Students should recognize that a right angle is 90 degrees and that it has a symbol that represents it. Students should realize that	Students should realize through manipulative activities that the distance is the same everywhere between two parallel lines. No mathematical proof is required.	triangular plate, ruler	s-II-3

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	two lines that are perpendicular to the same line are parallel to each other and that the distance between two parallel lines is the same everywhere. They should be able to construct perpendicular lines as well as parallel lines.			
S-4-6	**Congruence of plane figures:** Mainly through manipulative activities. Congruence refers to two figures having the same size and shape. They can be made to coincide with one another under translation, rotation and flipping over. Congruent figures have the same corresponding angles and edges.	Students can observe many examples of congruence on figures embedded with rich translational or rotational modes of symmetry (such as in a logo design.) Notice that translation, rotation and flipping over are regarded as modes of operation, and not as formal terms of geometrical transformation. Formal introduction of these terms are not required.	Symmetrical figures to demonstrate translation and rotation	s-II-2
S-4-7	**Triangle:** Instruction can familiarize students to special triangles through their characteristics of angles and sides, including equilateral triangle, isosceles triangles, right-angled triangles, scalene triangles and obtuse triangles. Students should be able to construct these triangles.		Various kinds of triangles	s-II-3
S-4-8	**Quadrilateral:** Instruction can familiarize students to special quadrilaterals through their characteristics of angles and sides, including squares, rectangles, parallelograms, rhombuses and trapeziums. Students should be able to construct these quadrilaterals.	Construct various quadrilaterals, including squares, rectangles and parallelograms.	Various kinds of quadrilaterals	s-II-3
R-4-1	**Converting a two-step problem into a single**	Limited to whole numbers. Students have already		r-II-3

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	expression: Solving a multiple-step problem in a single expression is one of the important foundations to the learning of algebra. Instruction should introduce the convention about the order of operations (carrying out from left to right, multiplication and division before addition and subtraction, and expressions in parentheses are evaluated first). Instruction should help students understand how to reduce the number of terms step-by-step.	learned to solve two-step problems (N-2-8, N-3-7) in the second and third grades. They should be reviewed first before helping students to learn how to solve two-step problems in a single expression.		
R-4-2	**Rules of the four operations of arithmetic (I)**: This entry will cover the rules for two-step computations, including rules for the mixed modes of addition and subtraction, and of multiplication and division. The focus is on applying the computational properties of numbers in computation.	In relation to addition and subtraction, there is no need to simplify by eliminating the parenthesis in expressions of the form $a-(b-c)$. Multiplication and division will be confined to three numbers in a row and the realization that different orders of multiplication gives the same product, and that multiply first then divide gives the same answer as divide first then multiply. Instruction should include special examples to illustrate the following principle: "After converting application problems into computational expressions which are further modified through the rules of computation as specified in this entry, the expressions cannot be interpreted in terms of the original problem contexts. "		r-II-4
R-4-3	**Using words to represent mathematical expression:** Students should understand that mathematical formulae are composed of words and	Examples include the area of a rectangle = length × width and the perimeter of a square =length × 4, as		r-II-5

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	operations and be able to apply them on the right occasion. This entry can combine with other instructional activities (for example, S-4-3).	in S-4-3.		
R-4-4	**Number pattern and reasoning (II):** Mainly through manipulative activities on observing and reasoning about patterns of changes in two dimensional tables and figures. Instruction should help students understand the concepts of odd and even numbers, and their properties with respect to addition, subtraction and multiplication.	The purpose of this entry is to motivate students through activities to apply simple reasoning and explanation on number patterns that are found in 10×10multiplication table, calendar, etc. No formula is required as it is not the purpose of this entry.		r-II-2
D-4-1	**Reading off from charts and line graph together with the construction of charts:** Reading off and describing data from charts and line graphs in everyday life. Instruction can go hand in hand with other topics and include the construction of charts.	Both instruction and examples should pay attention to the differences between data for classification and data that carry ordinal information.		d-II-1
Grade 5				
N-5-1	**Place value of the decimal system:** From a trillion to a thousandth. Instruction should help students integrate the concepts of whole number and decimal. Students should understanding that very large and very small numbers can also be represented through the use of the decimal system.	Students should be proficient in treating problems that involved "multiplied by 10" and "divided by 10" in the decimal system, together with treatment of their extended problems, such as 300×1200 and $600000 \div 4000$.	Decimal system table (from quadrillion to thousandth)	n-III-1
N-5-2	**Problem solving: Multi-step application problems**. Application problems should involve at most three-step except for taking	This entry requires expressing terms in a single expression. Instruction should, in principle, use problems that are familiar		n-III-2

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	averages, which may involve more steps.	to students and can be solved by directly listing terms in a single expression. Coverage should involve problems that require a three-step procedure within a distributive context so that it can be connected with the instruction on the distributive rule (R-5-2).		
N-5-3	**Common factors and common multiples:** Coverage include factors, multiples, common factors, common multiples, greatest common factor, least common multiple, together with their meanings.	Instruction mainly focuses on conceptual understanding of common factors and common multiples. Short division is not required for this entry (N-6-1, N-6-2).		n-III-3
N-5-4	**Fractions with different denominators:** Instruction should include operating with equivalent fractions by reducing and expanding of fractions as well as their comparisons. Instruction should help students understand how to do addition and subtraction of fractions with different denominators through reduction to a common denominator, and nurturing in students the habit of always reducing fractions for ease in computation.	It is not suggested to find the common denominator for two or more fractions by directly multiplying their denominators. Regarding the reduction to a common denominator in the fifth grade, it can be done through anyone of the following three ways. First, the denominators should be single digit. Second, one of the denominators is a multiple of the other, and with both denominators being less than a hundred. Third, the common multiple of the two denominators can be readily found by multiplying by 2, 3, 4 or 5 (for example, 12 and 18).		n-III-4
N-5-5	**Multiplication of fractions:** Coverage includes the meaning of multiplying a whole number by a fraction, and a fraction by another fraction. Students should know how to use fraction reduction to	Instruction should build up the connection between the meaning of terms like “half of” and “1/2 times”.		n-III-6

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	simplify multiplication, and that the commutative rule of whole numbers also holds for fractions through the formula of fraction multiplication. Instruction should clarify the misconception that the product must be bigger than the multiplicand.			
N-5-6	**Fractional representations of whole numbers divided by whole numbers:** Instruction should include explanations and the rationality in regarding the division of two whole numbers as fractions through the concepts of equal distribution (in measurement) and fair sharing.	The difficulty of this entry is in conceptual understanding rather than in computation. Instructors should actively help students to overcome the fixed impression that division of whole numbers will always involve a remainder, and help them understand the rationality of regarding the quotient as a fraction. The concept of quotitive division should be connected to the concept of ratio (N5-10).		n-III-5
N-5-7	**Fractions divided by whole numbers:** Instruction should include an explanation about the meaning of dividing fractions by whole number and eventually leads to transforming the questions into multiplying by unit fractions.	Partitive division can be taught by appealing to former experience in treating multiplication by fraction as fractional multiples (N-5-5). Quotitive division can be taught together with the concept of proportion (N5-10).		n-III-6
N-5-8	**Multiplication of decimals:** Instruction should include an explanation about the meaning of multiplying whole numbers by decimals and multiplying decimals by decimals. Instruction should cover the vertical format of multiplication when the multiplier is a decimal, as well as the rationality behind the	Instruction can start by connecting the meaning of "multiplied by 0.1, " "multiplied by 0.01, " and also former experiences of multiplication in the vertical format and then make extension of the results. The multiplication by decimals in vertical format is similar to that of whole numbers, yet it must be made clear that the way		n-III-7

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	vertical format by means of the concept of place value. Instruction should clarify the misconception that the product must be bigger than the multiplicand.	the decimal point is taken in multiplication is different from that in addition.		
N-5-9	**Whole numbers and decimals divided by whole numbers (quotient is in decimal format):** Instruction should include the meaning of having whole numbers divided by whole numbers (quotient is in decimal format) and decimals divided by whole numbers. Instructors should use the concept of place value to explain the rationality of division in the vertical format. Students should be able to take approximates of quotients in case of indivisibility, and be proficient in finding the decimal forms to proper fractions with 2, 4, 5 and 8 as the denominators.	This entry only attends to division problems with quotients to three decimal places. It is a good idea to let students find out for themselves the problem of indivisibility and repeating decimals, an then the instructors can step in by introducing how to take approximates for this kind of problems (N-5-11). It is not necessary to teach students how to name repeating decimals.		n-III-7
N-5-10	**Problem solving: Application of ratios.** Instruction should include the division between whole numbers, together with the introduction to such terms as percentage, discount and percent sales off.	This entry is limited to applied contexts in which the sum of the proportions is not greater than 1 (100%). For situations where in which the sum of the proportions is greater than 1 is postponed to N-6-8.		n-III-5 n-III-9
N-5-11	**Problem solving: Taking approximates of decimals.** Instruction should include real life application and the method of rounding. Students should know what to make of the quotient under indivisibility as well as the meaning of approximation.	It is not necessary to mention the terms “error” and “approximate” during instruction.		n-III-8

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N-5-12	**Area: Units in acre, hectare, and square kilometer.** Instruction should include real life application of area, including conversion among units and to square meters, as well as the use of approximates.	The focus of this entry is on the larger units of area. Instructors should use daily life examples that are familiar to students so that they can have a sense of the sizes of the units.		n-III-11
N-5-13	**Weight: Unit in tonne.** Instruction include real life application of weight, conversion between tonne and kilogram, and the use of approximates.	Instructors should use everyday examples that are familiar to students so that they can have a quantity sense of the units.		n-III-11
N-5-14	**Volume: Unit in cubic meter.** Instruction should include simple measurement, a sense of the size, estimation and computation.	Computation should not involve compound number for this entry. The conversion from 1 cubic meter to 1 cubic centimeter is a little complicated and is not required for assessment.		n-III-11
N-5-15	**Problem solving: Volume of an empty container, capacity, and volume.** Instruction should include an explanation of the relationship between volume and capacity. Students should know the meaning behind liquid volume.	Instruction should include how to find the volumes of irregular objects by appealing to the concept of capacity.		n-III-12
N-5-16	**Problem solving: Problems related to the multiplication and division of time.** Instruction should cater to solving time-related problems within the coverage specified for fractions and decimals	This entry includes amounts of time expressed in fractions and decimals, such as 15 minutes expressed as a quarter-hour (15/60=1/4), and 1/5 of an hour is 12 minutes (60×1/5=12). It is permissible to include work rate problems for this entry.		n-III-11
S-5-1	**Properties of triangles and quadrilaterals:** Through operative activities and simple reasoning, instruction should include the interior angles of a	Students should be able to conduct simple reasoning, such as the sum of the vertices of a quadrilateral is 360 degrees and that there cannot be two obtuse		s-III-5

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	triangle sum to 180 degrees, the sum of any two sides of a triangle is greater than the third, and that the opposite sides and angles are equal for a quadrilateral.	angles in a triangle.		
S-5-2	**Areas of triangles and quadrilaterals:** Through operative activities and simple reasoning, instruction should build up an understanding of the area formulae through dissection and recombination and then apply them.	Coverage include computation of area problems. Should fractional or decimal lengths be used, they should be presented after instruction of the multiplication of fractions and decimals,	Triangle, quadrilateral	s-III-1
S-5-3	**Sector:** Introduce the definition of sector and the meaning of angle at center. Instruction should point out that a sector is a part of a circle and hence it has a fractional relationship with the circle (it can be viewed as how many parts of a circle). Students should be able to draw a specified sector.	Coverage should include sector with angle at center bigger than 180 degrees. Students should understand that sector with angle at center equal to 90 degrees is the same as a quarter circle and other similar situations. Specified sectors to be drawn should include "to draw sectors of 1/3 and 1/6 of a circle for a given circle " and "to draw a sector for a given radius and angle at center".	Circle, sector	s-III-2
S-5-4	**Line of symmetry:** Enable students to understand the meaning of axis of symmetry, points of symmetry and angles of symmetry. Instruction should include operative activities so that students can find out the line symmetrical properties of special plane figures. Students should be able to make simple reasoning based on line symmetry, and produce or draw line symmetrical figures.	Instruction should include operative activities so that students can observe the properties of such symmetrical figures as equilateral triangles, isosceles triangles, squares, rectangles, rhombuses, and kites (of which the figures are introduced but not their names). During instruction, it is suggested that the lines of symmetry are to be presented as perpendicular or parallel to the edges of the paper (yet this condition is not required in operative activities).	Line symmetrical figures, scissors, graph paper, plane figures	s-III-6

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
		Coverage can include lines of symmetry of polygons.		
S-5-5	**Cube and cuboid:** Instruction include the computation of the volumes and surface areas of cubes and cuboids, and be familiar with their formulae.	Students can compute the surface areas of cuboids, but without the requirement to commit the formulae to memory.	Unit cube	s-III-4
S-5-6	**The relationship between planes in 3-dimensional space:** Mainly through operative activities, instruction should include the perpendicular and parallel relationship observed between planes in daily life phenomenon, the perpendicular or parallel relationship among planes in a cube (or a cuboid). Students should be able to use a given cube (or a cuboid) to check the perpendicular or parallel relationship among given planes.	Instruction should emphasize on the rationality of the operations that correspond to the symmetry behind, and not on the setting up of a rigorous definition. ~~It is suggested to avoid using triangular plate for the purpose of this entry because students are easily confused its proper uses.~~ (Last sentence can be skipped without translation)	Cube, cuboid, cylinder, cone	s-III-3
S-5-7	**Sphere, cylinder and prism:** Instruction should include the recognition of spheres, (right) cylinders, (right) pyramids and (right) cones mainly through operative activities. Students should recognize the key components of cylinders and prisms as well as their nets. They should be able to tell whether the two bases of a cylinder are parallel, whether the side surface of a cylinder is perpendicular to the base and that the side surfaces of a pyramid and a cone is not perpendicular to the base.	Students should know that a cross-sectional plane of a sphere will give the locus of a circle (the terms cross-sectional plane and locus are not required in instruction) and terms like the center of a sphere and its radius. It is also not required to mention the terms “right” and “regular” while discussing solid figures in classrooms.	Two half-sphere (with center and radius shown), cylinder (tall-thin, short-fat), prism, pyramid, net	s-III-3
R-5-1	**Three-step procedure in a single expression:** Enable students to establish the habit of listing computation	It is reminded that learning how to express all computation into a single expression does not mean		r-III-1

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	procedures into a single expression, with an emphasis on the three-step procedure. Arithmetic mean can be introduced according to this approach. This entry is connected to the distributive rule.	that subsequent instruction on problem solving have to be done this way at this stage (N-6-9).		
R-5-2	**Rules of the four operations of arithmetic (II)**: This entry will cover mixed computations of multiplication and division. Coverage will include the distribution rule of multiplication over addition and subtraction. The emphasis is on applying operation rules so as to simplify computations. Students should be proficient in handling computations involving the four operations.	Mixed computations of multiplication and division: Instruction should include "a number divided by two numbers in a row is equal to a division by the product of the two numbers", but exclude the cancelation of parenthesis in the expression $a \div (b \div c)$. Coverage must include examples according to the following principles, namely, after transforming an application problem into an expression, steps should be taken to use operation rules to modify the expression before computation. Students should realize that the modified expression may no longer be explainable according to the original context.		r-III-1 by two
R-5-3	**Using symbols to represent mathematical formula:** This entry serves as initial experiences of algebra prior to junior high school. Instruction should provide students with initial perception of how symbols are used, and the implicit meaning of "having symbols to represent numbers" and "the combination of symbols with computational operators". This entry	It is suggested that this entry may be easier to implement via the area or volume formula of geometrical figures, or through the multiplication of fractions. This entry is not supposed to replace "using words to represent formula" (R-4-3), since the latter has the advantage of easier to understand.		r-III-3

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	should be combined with instruction in other units.			
D-5-1	**Construct line graphs:** Construct line graphs within daily life context.	Discern when to use line graphs for various purposes.		d-III-1
Grade 6				
N-6-1	**Prime numbers less than 20 and prime factorization of numbers:** Instruction should include prime numbers less than 20 together with their composite numbers. Coverage should also include principles in determining if a given number has 2, 3 or 5 as a prime factor as well as how to use the short division to carry out prime factorization.	Prime factorization instruction at this stage is suggested to limit to numbers that have an extra factor other than 2, 3, 5 and/or their powers, and that this extra factor should be either 11, 13, 17 or 19 (but 49, 77 or 91 is allowed).		n-III-3
N-6-2	**Greatest common factor and least common multiple:** Instruction should include how to find the greatest common factor and the least common multiple by the methods of prime factorization and short division. Coverage should also include the concept of relative prime between two numbers and applications to the reduction of fractions and the reduction to a common denominator.	This entry does not require finding the greatest common factor and the least common multiple from three or more numbers. Practices should include problems that require the reduction of fractions.		n-III-3
N-6-3	**Division of fractions:** Coverage should include the meaning of dividing whole numbers by fractions and fractions divided by fractions. Students should	This entry allows the omission of how to handle the remainders from division. If included, only reasonable real-life context should be used. No		n-III-6

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	understand that dividing by a number is equivalent to multiplying by its reciprocal.	assessment is required in the latter case.		
N-6-4	Division of decimals: Coverage should include the meaning of dividing whole numbers by decimals and decimals divided by decimals. Division is performed in the vertical format. Instructors should explain the rationality behind division under the vertical format by means of the concept of place value. Instruction should take care of the misconception that the quotient would be smaller than the dividend.	This entry allows the omission of how to handle the remainders from division. If included, only reasonable real-life context should be used and the quotient should be whole numbers. Care must be asserted in how to handle the remainder under the vertical format of division. No assessment is required on problems with remainders from division.		n-III-7
N-6-5	**Problem solving: Application problems involving the four operations on whole numbers, fractions, and decimals.** Instruction should include how to solve two- to three-step application problems and the use of approximates to help solve problems.	The coverage should include techniques in handling mixed modes multiplication and divisions that involve fractions and decimals.		n-III-2 r-III-2
N-6-6	**Ratio and the value of ratio:** Instruction should include ratios of terms with different units and those with the same unit. Students should understand the concept of proportion that involves the equivalence of two kinds of multiples (the foundation of proportional reasoning). Instruction should also include solving application problems in ratios and proportions.	All the numbers involved in setting up the ratio should, in principle, be whole numbers. However, simple fractions and decimals are permissible.		n-III-9
N-6-7	**Problem solving: Speed.** Instruction should include explaining the meaning of ratios, their values and their	Other than finding the average speed from different speeds in separate time intervals, all speed		n-III-9

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	applications as well as the meaning of speed. Students should be able to make conversion by changing larger units to smaller units. Coverage should include the concept of finding the average speed from different speeds in separate time intervals and also the formula "distance＝speed × time". The major concern is to nurture students the ability to carry out proportional reasoning.	problems at the primary level should assume a constant speed situation. Coverage should include problems with the context that given a constant speed, then the multiple amount of distance travelled is proportional to the multiple amounts of time spent. "Conversion by changing larger units to smaller units" refers to only changing larger units in time and distance to smaller units, respectively.		
N-6-8	**Problem solving: The antecedent and consequent terms.** Instruction should explain the meaning of a ratio and the application of the value of a ratio, including the exchange of the role of the antecedent and consequent terms.	An example of exchanging the roles of the antecedent term with the consequent term is to set a father's height as 1 unit and the son's height as 4/5 versus setting the son's height as 1 unit while the father's as 5/4.		n-III-9
N-6-9	**Problem solving: Using the relationship among numerical quantities to list appropriate expressions for problem solution (same as R-6-4).** Coverage can include: first, more complex models (for example, classroom seat arrangement models); second, more complicated computation (for example, multiplication rule, addition rule or their combined form); third, more complicated contexts (for example, the age problem, the river velocity problem, the sum and difference problem and the chicken and rabbit problem). This entry connects with R-6-2 and R-6-3.	Multiplication rule refers to problems such as finding out the number of combination in matching 3 shirts to 5 dresses. Addition rule refers to problems such as finding out how many numbers can be created out of using the numbers 1, 2 and 3, allowing repetition, to construct a two-digit odd number. Both the multiplication and addition rules are intended to help students to learn beginning ideas in enumeration. This entry is not intended to teach how to solve complicated enumeration problems and does not require listing out all terms in a single expression.		n-III-10 r-III-3

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
S-6-1	**Expansion and contraction:** Instructors should enable students to apply proportional thinking to understand the concepts of "diagram expanded by how many times" or "diagram reduced by how many times. " Students should know that under expansion and contraction, the corresponding angles are equal and the corresponding edges are in proportion.	Students should know that when a common plane figure is expanded or contracted in equal proportion, the resulting figure is within the same equivalence class (including circle), and that they should be able to explain the reason behind		s-III-7
S-6-2	**Problem solving: Map scale.** Introduce the meaning, notation and application of a map scale. Understand that the ratio of the lengths of two line segments on the map is equal to the distances on the ground.	Coverage should include the relationship of lengths between two maps and clarify the common misconception that for two maps with different scales, the one with a larger denominator corresponds to a longer length".	Map	n-III-9 s-III-7
S-6-3	**Pi, circumference, area of circle and area of sector:** Instructors can use the idea of dissection to explain the formula for the area of a circle. Instruction should include computation of the arc length and the area of a sector. Students should know that the following three ratios are the same: first, central angle: 360; second, arc of sector: circumference; third, area of sector: area of circle.	Since pi will be taken as 3.14, instruction should pay more attention to conceptual understanding in the computation rather than in the tedious computation itself. Activities can be designed to illustrate how to understand and estimate the areas of irregular figures. No assessment, however, is required on the area of irregular figures. Only direct questions in relation to the area and arc length of a sector (for examples, finding area or arc length when given a certain fraction of a circle or a given angle at center) is required to be covered Reasoning in reverse or problems with too many steps are not required at	Circles in dissected form (to introduce the concept of area)	s-III-2

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
		this stage (and postponed to junior high school S-9-5).		
S-6-4	**Volume and surface area of cylinders:** This entry concerns cylindrical figures and prisms. Instruction should enable students to understand the formula of cylindrical volume as being base area × height through the use of simple cylindrical models. Students should be able to compute the volume of simple composite figures.	It is not necessary to discuss all sorts of figures before informing students of their volumes as commonly given by base area × height. Solid figures for this entry basically refer to triangular prism, quadrangular prism and cylinder. Composite figures are confined to a combination of two solid figures. Surface areas are only required for those solid figures with a circular, rectangular, right-angled or parallelogramical base, with special attention to the rationality of the side lengths of the base. It is suggested not to over assess students on the computation of surface areas. In addition, it is not required to compute surface areas of composite figures.	Cylinder (including hollow ones)	s-III-4
R-6-1	**Computation rules of numbers:** At the final phase of primary math education, students should know that (1) whole number, decimal and fractions are all numbers and employ the same set of computational rules, (2) for the computation and rules of multiplication and division of whole numbers, they are easier to understand via the factorized approach, (3) gradually realize that multiplication and division are equivalent operations. The implementation of this entry can be immersed in other learning activities.	Students should understand various cancelation practices that can be used when solving mixed computation problems on decimals and fractions. Though multiplication and division carry different meaning when solving real-life problems, they are the same, however, computationally speaking.		r-III-2

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
R-6-2	Relationship of quantities: This constitutes the prior experience to algebra and function. Instruction can start from activities on concrete contexts or quantitative models through making observations, inferences and explanations.	Can use tables or statistical graphs to help identify the underlying patterns in the data. Can use simple formula to explain the relationship among quantities.		r-III-3
R-6-3	**Representations of quantitative relationship:** This constitutes the prior experience to algebra and function. Learn how to use texts and symbols to express the quantitative relationships into expressions in accordance to the underlying relationship of quantities within concrete contexts or models.	An example of a representation of quantitative relationship is to express the relationship between the lengths of daytime and nighttime as "length of daytime + length of nighttime =24. " This entry is connected with R-6-2 and includes instructing partial usage of symbols that paves the way to learn "using symbols to represent numbers" and the concept of "unknown" in junior high school. The main focus is on the expression of a relationship, and not on the abstract operation on algebraic symbols.		r-III-3
R-6-4	**Problem solving: Using the relationship among numerical terms to list appropriate expressions for problem solution (same as N-6-9).** Coverage can include: first, more complex models (for example, classroom seat arrangement models); second, more complicated computation (for example, multiplication rule, addition principle or their combined form); third, more complicated contexts (for example, the age problem, the river velocity problem, the sum and difference	The purpose of complex problem solving is to nurture thinking skills, not so much on formally listing all the steps in a single expression. See remarks to N-6-9.		r-III-3 n-III-10

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	problem and the chicken and rabbit problem)			
D-6-1	**Pie chart:** Read off, explain and construct pie charts in real-life context, including pie charts that are already divided into 100 sections and provided to students in learning activities.	Treat data that contain part whole relationship. Discern when to use various kinds of statistical diagrams.	Pie chart with a hundred divisions (draw pie chart in percentages)	d-III-1
D-6-2	**Problem solving:** Likelihood. Answer questions related to possibility by referring to statistical data. Preliminary experiences of probability, including the concept of quite likely, very unlikely, A is more likely than B.	Limit to a comparison of two events to establish the concept of "A is more likely than B". This entry is not a formal instruction on classical probability theory.		d-III-2
Grade 7				
N-7-1	**Prime numbers within a hundred:** Introduce the definitions of prime number and composite number, and the use of sieve method to identify prime numbers.			n-IV-1
N-7-2	**Standard form of prime factorization:** Instruction should introduce the standard form of prime factorization and then apply it to solve problems involving multiples and factors.			n-IV-1
N-7-3	**Negative numbers and the four operations (including fractions and decimals):** Instruction should introduce how to use positive and negative signs to represent quantities in real-life context. Coverage include the concept of opposite number (i.e. additive inverse) and computational problems involving the four operations on positive and negative numbers.			n-IV-2

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
N-7-4	**Operation rules of numbers:** Coverage should include the commutative, associative and distributive rules, together with the operation rules of $-(a+b)=-a-b$ and $-(a-b)=-a+b$.			n-IV-2
N-7-5	**Number line:** Number line will be extended to include negative numbers. Instruction should include comparison of the sizes of numbers, the meaning of absolute values and the representation of the distance between two points a and $b$as $\|a-b\|$ on the number line.	Absolute value is introduced to denote the distance between two points on the number line. It is not necessary to treat equations and inequalities involving absolute values for this entry.		n-IV-2
N-7-6	**Meaning of an exponent:** An exponent is the power of a non-negative whole number, with the special case when $a \neq 0$, then $a^0=1$.Coverage should include comparison of sizes of two numbers with the same base and rules of computation for exponents.			n-IV-3
N-7-7	**Exponential rules:** Instruction can use numerical examples to introduce the exponential product rules to numbers of the same base, namely, $a^m \times a^n = a^{m+n}$, $(a^m)^n = a^{mn}$、$(a \times b)^n = a^n \times b^n$, where m and n are non-negative whole numbers. Likewise, the exponential quotient rules of the same base can be introduced through numerical examples ($a^m \div a^n = a^{m-n}$, where m and n are non-negative whole numbers with $m \geq n$).			n-IV-3

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
N-7-8	**Scientific notation:** Coverage should include using scientific notation to represent positive numbers, which can be very large (with its power being a positive whole number) or very small (with its power being a negative whole number).	This entry aims at enabling students to understand the meaning and application of scientific notation. For example, students should understand that 1 nanometer is equal to 10^{-9} meter. As regards the negative power involved, it can be introduced by way of converting from the decimal format. This entry does not involve other base nor the four operations on numbers in the scientific notations.		n-IV-3
N-7-9	**Ratio and proportion:** Coverage should include such concepts as ratio, proportion, direct proportion, inverse proportion together with their operations and applications. It is suggested that meaningful values of ratios and proportions be used in instructional context.	This entry does not involve mixed fractions. Division is the expected operation when the context calls for a ratio of two fractions.	Calculator	n-IV-4 n-IV-9
S-7-1	**Simple figures and geometric symbols:** Coverage should include introduction to the concepts of point, straight line, line segment, ray, angle, triangle as well as their symbols. #			s-IV-1
S-7-2	**Three view drawings:** Coverage should include the concepts of front view, top view and side (left or right) view of solid figures. These solid figures must not be hollow and should be embedded within a cube of dimension 3×3×3.		Blocks	s-IV-16
S-7-3	**Perpendicularity:** Coverage should include the symbol for perpendicularity, the perpendicular bisector of a line segment and the			s-IV-3

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	meaning of the distance from a point to a straight line.			
S-7-4	**Properties of line symmetry:** Coverage should include such properties as symmetric line segments are of equal length, symmetric angles are equal and the line segment that joins two symmetrical points is bisected by the line of symmetry.			s-IV-5
S-7-5	**Basic figures that are line symmetric:** Coverage should include such line symmetric figures as isosceles triangle, square, rhombus, kite and regular polygon.			s-IV-5
G-7-1	**Cartesian coordinate system:** Instruction should cover the meaning of a plane rectangular coordinate system and related terminologies (abscissa, ordinate and quadrant), together with the use of distance and orientation to specify positions in a plane rectangular coordinate system.			g-IV-1
A-7-1	**Algebraic symbol:** Instruction should include using algebraic symbols to present commutative rules, distributive rules and the associative rule, the simplification of linear expressions and similar terms as well as using symbols to represent mathematical problems in real life context.			a-IV-1
A-7-2	**Meaning of a first degree equation with one unknown:** Instruction			a-IV-2

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	include explanation regarding the meaning of a first degree equation with one unknown together with its solution. Students should be able to list a first degree equation with one unknown from a realistic context.			
A-7-3	**Solution to a first degree equation with one unknown and its application:** Instruction should include the axiom of equality, the concept and rules of moving terms to the other side of the equation, the idea of checking the solution and also solving application problems.			a-IV-2
A-7-4	**The meaning of simultaneous linear equations with two unknowns:** Instruction should include explanation regarding the meaning of simultaneous linear equations with two unknowns and their solutions. Students should be able to list simultaneous linear equations with two unknowns from a realistic context.			a-IV-4
A-7-5	**Methods to solve simultaneous linear equations with two unknowns and its application:** Instruction should include the method of elimination by substitution and elimination by addition or subtraction as well as solving application problems.			a-IV-4
A-7-6	**Geometric meaning of simultaneous linear equations with two unknowns:** Instruction should include the meaning			g-IV-2 a-IV-4

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	and differences among the graphs of $ax + by = c$, $y = c$ (horizontal line) and $x = c$ (vertical line). It is only required to seek the solution when the simultaneous linear equations intersect at a point.			
A-7-7	**The meaning of a first degree inequality with one unknown:** Instruction should explain the meaning of inequality and list a first degree inequality with one unknown from a realistic context.			a-IV-3
A-7-8	**The solution of a first degree inequality with one unknown and its application:** Coverage should include the solution to a single first degree inequality with one unknown, marking out the range of the solution on a number line and solving application problems.			a-IV-3
D-7-1	**Statistical graphs and tables:** Instruction should include the collection of data from daily life events and then organize them to produce graphs or tables that contain the raw data or their proportions. Graphs covered should include histograms, bar charts, pie charts, line graph and contingency tables. Students should know how to use calculators when the data set is more involved. Instructors can use computer software for demonstration purposes.		Calculator	d-IV-1 n-IV-9
D-7-2	**Statistical data:** This entry focuses on using the mean, median and mode to describe the central		Calculator	n-IV-9 d-IV-1

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	tendency of a data set. Calculators with the M+ and the Σ buttons should be used to compute the mean.			
Grade 8				
N-8-1	**Square root:** Instruction should explain the meaning of square root, simplification of radicals and the four operations on radicals.	It is permissible to use expressions of radicals that can be simplified by the multiplication formula without mentioning the latter until they are covered in A-8-1.		n-IV-5
N-8-2	**Approximation from square root:** Instruction should include how to obtain an approximate to the square root of a given number, the whole number portion of a square root, how to carry out digit-by-digit calculation and how to use the square root button $\sqrt{}$ on a calculator.	The whole number portion of the square root of a given number can be estimated either by way of geometry, digit-by-digit calculation or by using a calculator.	Calculator	n-IV-6 n-IV-9
N-8-3	**Recognition of a sequence:** It is suggested that students can be introduced to the concept of a sequence through examples from daily life together with their pattern of regularity, including those from geometric figures.			n-IV-7
N-8-4	**Arithmetic sequence:** Instruction should include an explanation about the meaning of an arithmetic sequence and how to find the general term when given the first term and the common difference.	This entry does not require the treatment of the following kinds of problems: Find the first term, the difference term or the sum of a sequence when given two other non-consecutive terms of the sequence, such as "Find the first term, the difference term or the sum when given the values of a_5 and a_9."		n-IV-7
N-8-5	**The sum of an arithmetic series:** Instruction should include the formula for the	This entry does not require finding the first term, the number of terms or the		n-IV-8

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	sum of a series and application problems encountered in daily life.	difference term when given the sum of a series.		
N-8-6	**Geometric sequence:** Instruction should include an explanation about the meaning of a geometric sequence, and how to find the common term when given the first term and the common ratio.	This entry does not require the treatment of the following kinds of problems: Find the first term, the number of term or the common ratio when given two other non-consecutive terms of the sequence, such as "Find the first term, the difference term or the sum when given the values of a_5 and a_9. "		n-IV-7
S-8-1	**Angles and their various kinds:** Instruction should include explanation about various relationship between two angles (supplementary angles, complimentary angles, vertically opposite angles, corresponding angles, alternate angles, interior angles) as well as the meaning of an angle bisector.			s-IV-2
S-8-2	**The sum of interior angles of a convex polygon:** Instruction should include explaining the meaning of a polygon, its interior and exterior angles, the formula for the sum of the interior angles and the angle at the vertex of a regular n-polygon.	This entry does not involve the formula for the sum of the exterior angles of a polygon.		s-IV-2
S-8-3	**Parallel lines:** Instruction should include explaining the meaning of parallel lines and the symbol for lines being parallel, properties of the angles of intersection and that the distance between parallel lines is equal everywhere.			s-IV-3
S-8-4	**Congruent figures:** Instruction should include			s-IV-4

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	explaining the meaning of congruent figures (i.e. two figures completely coincide after translation, rotation or flipping). Furthermore, when two polygons are congruent, their corresponding sides and angles are equal to each other, and vice versa.			
S-8-5	**Congruency of triangles:** Coverage include the criteria in determining congruency (SSS, SAS, ASA, AAS, RHS) and the symbol that denotes congruency (≅).			s-IV-9
S-8-6	**Pythagoras theorem:** Coverage should include an explanation of the meaning and history of the Pythagoras theorem (also known as the G*ougu* theorem or the *Shang-Gao* theorem), its application in problems encountered in daily life, and the converse of Pythagoras theorem.			s-IV-7
S-8-7	**Area of plane figures:** Instruction should include the height of an equilateral triangle, the formula for its area, and the areas of composite figures.			s-IV-8
S-8-8	**Basic properties of triangles:** Instruction should include the properties that the base angles of an isosceles triangle are equal, angle opposite to a larger side is larger and vice versa for a non-isosceles triangle, the sum of two sides of a triangle is larger than the third, and the exterior angle is equal to the sum of the two interior opposite angles.		protractor	n-IV-4 s-IV-9

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
S-8-9	**Basic properties of a parallelogram:** Instruction should include the geometrical properties concerning the angles, sides, and diagonals of a parallelogram.			s-IV-8
S-8-10	**Squares, rectangles, kites and their properties:** Instruction should include the geometrical properties concerning the equal length and equal bisection of the diagonals of a rectangle, the perpendicularity and equal bisection of the diagonals of a rhombus, and one of the diagonal of a kite is a perpendicular bisector of the other diagonal.			s-IV-8
S-8-11	**梯形的基本性質：**等腰梯形的兩底角相等；等腰梯形為線對稱圖形；梯形兩腰中點的連線段長等於兩底長和的一半，且平行於上下底。**Basic properties of a trapezium:** Instruction should include isosceles trapeziums are line symmetric figures, the base angles of an isosceles trapezium are equal, and the line joining the midpoints of the isosceles sides is equal to half the sum of the lengths of two bases and that it is parallel to the two bases.			s-IV-8
S-8-12	**Construction by straightedge and compass and geometrical reasoning:** Instruction should cover how to replicate a given line segment, a circle, an angle and a triangle. Students should be able to construct a perpendicular bisector, an		compass	s-IV-13

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	angle bisector, parallel line, vertical line, and also be able to list the geometrical properties that substantiate geometrical reasoning. #			
G-8-1	**Distance between two points in a rectangular plane coordinate system:** Students should understand that the distance between two points $A(a,b)$and $B(c,d)$ is found by using the formula $\overline{AB} = \sqrt{(a-c)^2+(b-d)^2}$ and be able to solve application problems encountered in daily life.			g-IV-1
A-8-1	**Formula for the multiplication of two binomials:** Formula covered will include the square of a binomial: $(a+b)^2 = a^2+2ab+b^2$, $(a-b)^2 = a^2-2ab+b^2$; and the multiplication rules for two binomials: $(a+b)(a-b) = a^2-b^2$, and $(a+b)(c+d) = ac+ad+bc+bd$.			a-IV-5
A-8-2	**The meaning of a polynomial:** Coverage should include explanation about the meaning of a polynomial with one unknown and relevant terms (polynomial, number of terms, coefficient, constant term, first degree term, second degree term, highest degree term, ascending and descending order).			a-IV-5
A-8-3	**The four operations of polynomials:** Instruction should include the addition and subtraction of polynomials in vertical and horizontal formats, multiplication of	This entry does not involve the method of detached coefficients.		a-IV-5

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	polynomials in vertical format (products should at most be up to degree three), and division in which the dividends are polynomials of degrees two.			
A-8-4	**Factorization:** Instruction should include the meaning of a factorial expression (confined to first degree factorial for a second degree polynomials) and the factorization of a second degree polynomial.			a-IV-6
A-8-5	**Techniques of factorization:** The techniques covered should include taking out the common terms, using the square of a binomial formula and the cross-multiplication technique.	This entry only deals with factorization of quadratic expressions $ax^2 + bx + c$ with whole number coefficients or those that can be factorized by formula for the multiplication of two binomials (A-8-1). This entry does not deal with homogeneous quadratic expressions or nonhomogeneous quadratic expressions but with a linear term in its coefficient.		a-IV-6
A-8-6	**The meaning of a quadratic equation with one unknown:** Instruction should include explanation about the meaning and solution to quadratic equation in one unknown and formulate a quadratic equation from a concrete context.			a-IV-6
A-8-7	**The solution and application of a quadratic equation with one unknown:** Instruction should include using factorization, completing the square and formula to solve quadratic equation in one unknown. Instruction should also include		Calculator	a-IV-6

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	application problems and how to use calculators to estimate the solution to a quadratic equation.			
F-8-1	**Linear function:** Instruction should use correspondence relationship to enable students to recognize the concept of function (without stating the abstract format of $f(x)$), the constant function $(y = c)$, and the linear function $(y = ax + b)$.			f-IV-1
F-8-2	**Graph of linear function:** Instruction should include graphs of constant functions and linear functions.			f-IV-1
D-8-1	**Handling statistical data:** Coverage should include cumulative frequency, relative frequency and cumulative relative frequency polygons.		Calculator	n-IV-9 d-IV-1
Grade 9				
N-9-1	**Continued proportion:** Instruction should include the reasoning and notation for continued proportion as well as the basic operations to handle continued proportion expressions and their application problems. Use calculator when complex numerical values are involved.		Calculator	n-IV-4 n-IV-9
S-9-1	**Similar figures:** Instruction should include the meaning behind expanding and contracting a plane figure, the similarity of polygons, the equivalence of angles and the proportionality of the lengths of corresponding sides.			s-IV-6

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
S-9-2	**Properties of the similarity of triangles:** Instruction should include criteria to determine the similarity of triangles (AA, SAS, SSS), the ratio of the lengths of corresponding sides is equal to the ratio of corresponding heights, the ratio of corresponding areas is equal to the ratio of the squares of the lengths of the corresponding sides, the symbol ~ to represent similarity, and the application of the concept of similarity to solve applied problems.			s-IV-10
S-9-3	**Property of proportional line segments intercepted between parallel lines:** Coverage includes the line connecting the midpoints of two sides of a triangle is parallel to the third side with its length being half of the third, properties of proportional line segments intercepted between parallel lines, the discernment if two lines are parallel based on whether the line segments intercepted between parallel lines are proportional, as well as its application to solve relevant problems.			s-IV-6 s-IV-10
S-9-4	**The invariance of the ratios of the sides in similar right-angled triangles:** This entry refers to the property that for a given acute angle in a series of right-angled triangle, the ratios of the sides is an invariant regardless of the size of the triangles. The three sides can be registered as $1:\sqrt{3}:2$ when the three	Angles are suggested to be confined to 30 degrees, 45 degrees and 60 degrees when the intention is for students to be familiar with the concept without the help of a calculator.	Calculator	s-IV-10 s-IV-12 n-IV-9

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	interior angles are 30°, 60°, 90°, respectively, and as $1:1:\sqrt{2}$ when the three interior angles are 45°, 45°, 90°, respectively.			
S-9-5	**Arc length and area of a sector:** Instruction should include using π to represent the ratio of the circumference to the diameter of a circle, the meaning of chords, arcs and circular segments, and also the formulae for the arc length and area of sectors.			s-IV-14
S-9-6	**Properties of circles:** Instruction should include an explanation about the relationship among angles at the center, angles at the circumference, and the measures of corresponding arcs, the property that the opposite angles are supplementary in an inscribe quadrilateral, and the two tangents from an exterior point to a circle are of equal length, etc..			s-IV-14
S-9-7	**Relationships between points, straight lines and circles:** Coverage includes the relative positions between a point and a circle (interior, exterior and on the circumference), the relative position between a straight line and a circle (non-intersecting, tangent, and intersecting at two points), the radius from the center to the tangent point is perpendicular to the tangent line (property of tangency), and the perpendicular line segment from the center to a chord is a perpendicular bisector of the chord			s-IV-14

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	(distance of chord from center).			
S-9-8	**The circumcenter of a triangle:** Instruction should include an explanation of the meaning of circumcenters and circumcircles, the property that the distances from the circumcenter to the three vertices are the same, and that the circumcenter of a right-angled triangle is the midpoint of the hypotenuse.			s-IV-11
S-9-9	**The incenter of a triangle:** Instruction should include an explanation of the meaning of incenters and inscribed circles, the property that the incenter is equidistant from the three sides of a triangle, the area of a triangle equals to the product of its perimeter multiplied by the radius of the inscribed circle divided by two, and that the radius of the inscribed circle is equal to half of the difference of the hypotenuse from the sum of the other two sides.			s-IV-11
S-9-10	**The centroid of a triangle:** Instruction should include an explanation of the meaning of the centroid and the medians of a triangle, the property that three medians divide the area of a triangle into six equal portions, the distance from the centroid to a vertex is twice that from the centroid to the midpoint of the opposite side, as well as the physical meaning of a centroid.			s-IV-11
S-9-11	**The meaning of mathematical proof:**	The teaching materials for proof should be around		s-IV-3 s-IV-4

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	Instruction should include an introduction to geometric reasoning (by providing the geometrical properties being based on for a certain claim) and algebraic reasoning (by providing the algebraic properties being based on for a certain claim).	those mathematical concepts specified in this manuscript and nothing beyond the present stage of learning.		s-IV-5 s-IV-6 s-IV-9 s-IV-10 a-IV-1
S-9-12	**Lines and planes in 3-dimensional space:** Coverage should include introducing schematic diagrams for cuboids and tetrahedrons, and then use them as case studies to illustrate the concepts of parallel, perpendicular and slanting relationship between lines and also the concepts of parallel and perpendicular between lines and planes.	The entry S-5-6 only covers the concepts of parallel and perpendicular between two planes. The medium of instruction is by way of manipulative activities. What is new in this entry is introduction to "the concepts of parallel, perpendicular and slanting relationship between lines and also the concepts of parallel and perpendicular between lines and planes. " Furthermore, the emphasis is on conceptual understanding.	Cuboid, tetrahedron	s-IV-15
S-9-13	**Surface area and volume:** Coverage should include the nets and surface areas of right prisms, right circular cones and regular pyramids as well as the volume of right prisms.	The entry S-6-4 only covers the volumes of vertical cylinders. This entry intends to review and then deepens students' understanding of this concept. Students should be able to compute the surface areas of cylinders and regular prisms through their nets.		s-IV-16
F-9-1	**The meaning of quadratic functions:** Instruction should include explaining the meaning of quadratic functions and expressing the relationship between two variables as a quadratic function in a realistic context.			f-IV-2
F-9-2	**The graph of a quadratic function and its extremum:** Instruction should include	This entry does not include the method of completing the square for quadratic		f-IV-2 f-IV-3

Code	Learning Content Heading and Description	Remarks	Teaching Aids	Corresponding Learning Performance
	introducing relevant terms to the quadratic function (axis of symmetry, vertex, highest point, lowest point, concave upward, concave downward, maximum value, minimum value), graphing the curves of $y = ax^2$, $y = ax^2 + k$, $y = a(x - h)^2$ and $y = a(x - h)^2 + k$, explaining the vertical line that passes through the vertex (highest point, lowest point) is the line of symmetry, explaining the relationship between the graphs of $y = ax^2$ and $y = a(x - h)^2 + k$ through translation, and finding the maximum or minimum value from a quadratic function that has gone through the process of completing the square.	equation nor solving application problems in quadratic function. These are topics for the tenth grade (F-10-1). The focus of this entry is on the properties of their graphs.		
D-9-1	**Spread of statistical data:** Coverage should include the concept of range, interquartile range and boxplot.	The entry D-7-2 focuses on one statistic (mean, median and mode) to represent a set of data. This entry focuses on using a boxplot to describe the central tendency of a set of data.	Calculator	n-IV-9 d-IV-1
D-9-2	**The recognition of probability:** Coverage should include the meaning of probability and the tree diagram (limit to only two layers).	Students should be able to exhaust all possibility through analysis by way of a tree diagram. At the junior high school level, the main focus will be on tree diagrams that are symmetrical (i.e. with equal number of nodes).		d-IV-2
D-9-3	**Classical probability:** Instruction should include finding probability of objects that are symmetrical (e.g. coins, dice, cards, balls in an urn) and exploring probability of objects that are not symmetrical (e.g. tossing tacks, throwing dice, casting "poe").		Calculator	n-IV-9 d-IV-2

Grade 10				
N-10-1	**Real number:** Instruction should include the number line, the meaning of decimals under the decimal system, trichotomy law, the characteristics of decimal expressions of rational numbers, the decimal estimation of irrational number (the proof that $\sqrt{2}$ is a rational number ★), and the operations of numbers in scientific notation.	★ This entry should include a definition of the number of significance digits in the scientific notation and pointing out that the number of significant digits should be retained after operations.	Calculator	n-V-1
N-10-2	**Absolute value:** Coverage includes equations and inequalities of absolute values.	It is suggested that discussion should, in principle, focus on inequalities of the forms $\mid x-a \mid > b$ and $\mid x-a \mid < b$, where the number b can be linked to the concept of error range in association to labels on commodities or industrial specifications. Symbols for intervals of real number can be introduced after obtaining solutions to inequalities, including those for the union of intervals and $\pm\infty$. They serve as representations for the solution ranges to inequalities and no operations on the set of ranges are required.		n-V-4
N-10-3	**Exponent:** Coverage should include explaining the meaning of decimal and fractional exponents of non-negative real numbers, geometric means and arithmetic-geometric mean inequality, reviews of exponential rules, meaning of real number exponents, and the use of the x^y function key in calculators.		Calculator	n-V-1
N-10-4	**Common logarithm:**	One intention of this entry	Calculator	n-V-1

	Coverage should include explaining the meaning of logarithm, the connection between common logarithm and scientific notation, and the use of the 10^x and log function keys in calculators.	is to reinforce through numerical operations the understanding that any positive number a can be rewritten as $10^{\log a}$. This entry does not involve any discussion of logarithm with a base other than 10.		
N-10-5	★# **Errors from numerical computation:** Coverage should include discussion about the limitation of using calculators, identifying the occurrence of numerical errors and taking an appropriate number of significant numbers. ★#		Calculator	n-V-2
N-10-6	**Sequence, series and recursive relation:** Coverage should include recursive series with finite terms, geometric series with finite terms, common summation formulae and mathematical induction.	This entry should, in principle, focus on treatment of recursive relationship to the first order and connect to the concepts of arithmetic and geometric series discussed at the junior high school level. Instruction should guide students to observe mathematical relations behind some patterns and then apply mathematical induction to prove their validity. The coverage here serves as an introduction to mathematical induction and that it should be immersed into the subsequent curriculum as well. Hence there is no need for excessive practices at this stage. Recursive relation can be used together with common logarithm to obtain approximate solutions to equations of the form $a^x = b$.		n-V-5
N-10-7	**Logic:** The purpose of this entry is to help students recognize what are prepositions and their negations, logical			n-V-6

	relationship between two prepositions (or, and, inference relationships), sufficient condition, necessary condition, and if-and-only-if condition. ★#			
G-10-1	**# Symmetrical properties of figures in a coordinate plane:** Coverage should include discussion about the symmetry with respect to the x axis, the y axis , the line $y = x$ and the symmetry with respect to the origin.	This entry does not require treatment regarding the general concepts of line symmetry and point symmetry.		g-V-2
G-10-2	**Equation of a straight line:** Instruction should include the concepts of slope, the meaning of slope in absolute value, the point-slope form, the translation of points and straight lines, equations for parallel and perpendicular lines, the distance from a point to a straight line, the distance between two parallel lines, and linear inequalities with two unknowns.	The content of this entry corresponds to the concepts of parallel and perpendicular lines that were covered in plane geometry at the junior high school level. This entry involves combined applications of the equations for parallel and perpendicular lines, including examples in finding the distance from a point to a straight line, the distance between two parallel lines, and the derivation of the area of OPQ when the coordinates of points P and Q are given.		g-V-4
G-10-3	**Equation of a circle:** Instruction should include the standard form of a circle.			g-V-4
G-10-4	**Straight line and circle:** Coverage should include the tangents to a circle as well as the algebraic and geometric determinant regarding the relationship between a circle and a straight line.	There is no need to deal with the relationship between two circles. This entry can use the notation of set in a descriptive sense, but only as a way to represent the solutions from inequalities without any further set operations on the intervals.		g-V-4
G-10-5	**Generalized angle and polar coordinate:**	Instruction should include the opportunity for students	Graph paper, protractor,	g-V-3

	Coverage should include the concept of a generalized angle and its terminal side, the definition of polar coordinates, and the conversion between the polar coordinates and the Cartesian coordinates by way of operations on graph paper.	to learn the material through operation. Introduction to this topic can start from angles within the range of $-180°$ to $360°$ and then extend to other angles as the occasions call for them. In addition, instruction should help students to understand the reason for assigning a direction to angles and also to understand that the slope of a line and the angle of between the line with the horizontal axis are equivalent concepts in a plane.	straightedge, compass	
G-10-6	**Trigonometric ratio:** Coverage should include definitions of the sine, cosine and tangent of an acute angle and then extends into the definitions of the sine, cosine and tangent of a generalized angle, together with their applications on special angles and the operations of the sin, cos and tan function keys on a calculator.	Instruction should provide students concrete experiences in estimating trigonometric ratios based on direct measurement from figures.	Graph paper, protractor, calculator	n-V-2 s-V-1 g-V-2
G-10-7	**Properties of trigonometric ratios:** Coverage should include sine law, cosine law, orthogonal projection, the connection between the slope of a line and the tangent of the oblique angle between a line and the positive horizontal axis, and using the arc sine, arc cosine and arc tangent function keys to compute the oblique angle or the angle between two intersecting straight lines, and trigonometric survey (＃).	Oblique angles are encouraged to be used as the concept image for angles discussed in this entry. In order to relate back to students' experiences of cuboid in junior high school, instructors can demonstrate trigonometric measurements and compute trigonometric ratios on truncated surfaces of a cuboid. Not only can this approach extend students' spatial concepts acquired in junior high school, but can be further applied to regular pyramids. The intention of this entry is to illustrate the properties of trigonometry, and so it is not necessary to	Calculator	n-V-2 s-V-1 g-V-3

		treat trigonometric survey as a separate unit, but immerse the discussion within the instruction context of trigonometric ratios that include major exemplary applications in history.		
A-10-1	**Operations of expressions:** Coverage should include the formulae for the trinomials, and operations on radicals and fractional expressions.			a-V-1
A-10-2	**Division rule of polynomials:** Coverage should include the factor theorem, the remainder theorem, dividing a polynomial by $(x-a)$, and expressing a polynomial in terms of $(x-a)$.	This entry focuses on synthetic division with $x-a$ as the divisor, with no requirement in extending it to the case of $ax-b$. Furthermore, this entry does not involve any treatment of the method of detached coefficients.		a-V-2
F-10-1	**Linear and quadratic functions:** Instruction should introduce the transition from equation to the $f(x)$ format, the relationship between the graph of linear function and that of $y=mx$,	Instruction should enable students to understand the necessity and rationality of the symbol $f(x)$ for function, including the relationship between the graph of $f(x)$with those of $f(x-h)$ and $f(-x)$. Coverage should also include applications of quadratic functions on closed intervals. Students should understand that the rationale behind the method of interpolation is the section formula	Calculator, graph paper	f-V-1 a-V-1 g-V-5
F-10-2	**Graphical characteristics of cubic function:** Coverage should include the symmetrical properties of quadratic and cubic functions, the global characteristics of the graphs of both functions are determined by the highest degree term, and the local characteristics of both graphs approaches a straight line.	Instruction should enable students to understand that the graph of any cubic function is the linear transformation of the graph for $y=ax^3+px$. Instruction can guide students to explore the linearity around a given point $x=h$ on a quadratic or cubic function.	Calculator, graph paper	f-V-2 a-V-1 g-V-5

F-10-3	**Polynomial inequality:** Coverage should include how to find the solution intervals for polynomial inequalities of first and second degrees or even higher-degree polynomials that are expressed in a factorized format. The solutions should then be related back to the graphs of the polynomials.	This entry offers a suitable occasion to introduce the interval notations for the set of real numbers after looking for the solution interval to a given polynomial inequality. Notations introduced can include the notation for the union of intervals and the $\pm\infty$ symbols. The interval notation can also be connected back to the descriptive format of set notation. Notice that this entry only requires using the interval notation to represent the solution to polynomial inequalities. No set operations on the intervals are required.		f-V-2 a-V-4
D-10-1	**Set:** Coverage should include the representation of a set and an introduction to the concepts of universal set, null set, subset, intersection and union of sets, complementary set, the belonging relation, the inclusion relation as well as Venn diagrams. ★ #	Besides allowing for experiences for students to use interval notation to express the solutions to polynomial inequalities, instruction can also connect set notation with concepts introduced in junior high school. One possible example is to discuss the relationship between various kinds of quadrilaterals in terms of set notation.		d-V-1
D-10-2	**Data analysis:** Coverage should include means and standard deviations for one dimensional data, scatter diagrams, best-fitting straight line and linear correlation for two dimensional data, as well as the standardization of data.	This entry should suitably reinforce the topic on data distribution graphs covered in junior high school, but with extended contexts. The concept of Students should understand that some statistics may be defined differently by different scholars or software. Accordingly, they should realize that different sources may produce different statistical values even for the same set of data, yet their interpretation are essential the same. In	Calculator	d-V-2 n-V-2 g-V-5

		addition, students should acquire the basic skill to decide how to choose suitable statistics based on the characteristics of the data. As regards the best fitting straight line, the key point to the instruction is to guide students to first identify if there exists any linear relationship among the data via inspection on a scatterplot, which is followed by discussing a suitable criterion to determine the case for "most fitting". It suffices to introduce this section using centered data so that the best fitting straight line will pass through the origin. Instructors should introduce this entry through common software and demonstrate how to use them to perform data analyses.		
D-10-3	**Systematic enumeration:** Coverage should include the methods of systematic exhaustion, tree diagrams, addition rule, multiplication rule, principle of inclusion and exclusion, as well as linear permutation and combination.	The coverage of permutation and combination in this entry is partly intended to serve the needs in teaching classical probability. This entry should also cover the expansion of binomials, thereby furnishing suitable examples for the application of combination.		d-V-6 d-V-7
D-10-4	**Classical probability of compound events:** Coverage should include sample spaces and events, properties concerning the classical probability of compound events and expected values.			d-V-3
Grade 11 (A strand)				
N-11A-1	**Radian measure:** Coverage should include the definition of radian measures, how to find arc	Instruction should include the interchange between the radian measures and the degree measures. It is	Calculator	n-V-7 n-V-2

	lengths and areas of sectors, as well as the operation of the rad function key on a calculator.	expected that further practices of the interchange between the two measures will be arranged within suitable learning contexts later on.		
S-11A-1	**The concept of space:** Coverage should include the basic properties of space, the relationship between two straight lines, between two planes, between a straight line and a plane with respect to their spatial positions, as well as a treatment of the theorem of the three perpendiculars.	Instruction should include a discussion about the concept of a dihedral angle. However, it is not necessary to deal with the general case of a dihedral angle geometrically other than for the right angle.		s-V-2
G-11A-1	**Plane vector:** Coverage should include the addition, subtraction and scalar product of vectors in a plane coordinate system as well as the concept of a linear combination of vectors.	Please note that this entry should refer back to relevant content covered in the tenth grade. The discussion is basically about position vectors, and the focus is on the concept of a linear combination of vectors.		g-V-1
G-11A-2	**Spatial coordinate system:** Coverage should include an introduction to the coordinates of points, the distance between two points and the projection from a point to the coordinate axes or to the coordinate planes.			g-V-1
G-11A-3	**Spatial vector:** Coverage should include the addition, subtraction and scalar multiplication of vectors in a space coordinate system as well as linear combinations of vectors.			g-V-1
G-11A-4	**Vector version of the triangular inequality:** Coverage should include the length of a vector and the triangular inequality for vectors.	Coverage should include a discussion of the triangular inequality for real numbers as a special case of the triangular inequality for vectors.		g-V-4 n-V-4
G-11A-5	**Trigonometric angles addition and subtraction formula:** Coverage should include the angle sum and the angle difference	Please note that this entry should refer back to the content covered in the tenth grade and focuses mainly on the formulas for sines		s-V-1 g-V-4

	formulas for sine and cosine, multiple angle formulas and half-angle formulas.	and cosines. The formula for tangent will serve as an exercise in derivation.		
G-11A-6	**Operations on plane vectors:** Coverage should include orthogonal projection and inner product of vectors, area formula and determinant, criteria to determine if two vectors are parallel or perpendicular, the angle between two vectors, and Cauchy's inequality.			g-V-5
G-11A-7	**Operations on spatial vectors:** Coverage should include orthogonal projection and inner product of vectors, the determination if two vectors are parallel or perpendicular, Cauchy's inequality and outer product.	Instruction can include a discussion about using Cauchy's inequality to derive the upper and lower bounds of the Pearson correlation coefficient for a set of data. ※		g-V-5
G-11A-8	**Third-order determinant:** Coverage should include using three vectors to determine the volume of a parallelepiped and the meaning and property of a scalar triple product.	One of the focuses of this entry is about the meaning of the volume of a parallelepiped.		g-V-5
G-11A-9	**Equation of a plane:** Coverage should include the normal vector and the standard form of the equation of a plane, the angle between two planes and the distance from a point to a plane.		Calculator	g-V-4 s-V-2
G-11A-10	**Equation of a straight line in space:** Coverage should include the parametric form and the symmetric equations of a straight line in space, the relationship between a straight line and a plane, the distance from a point to a straight line, and the distance between two parallel or skew lines in space.			g-V-4 s-V-2

A-11A-1	**System of linear equations in two unknowns in matrix format:** Coverage should include the notation of a matrix and the meaning of the linear combination formed by multiplying a matrix with a vector, the Cramer's formula, and a discussion about the conditions for having a unique set of solution, infinitely many sets of solutions or no solution.	This entry is concerned with concrete operations on vectors to understand the meaning of their linear combinations. The main focus of the Cramer's formula concerns the connection between the linear combinations of plane vectors with the areas of parallelograms.		g-V-4 a-V-3
A-11A-2	**System of linear equations in three unknowns:** Coverage should include solving the system of linear equations by the method of elimination and then by expressing them in matrix form, together with demonstrations of the use of computer software to solve a system of linear equations.	One possibility is to use polynomial interpolation as a way to generate a set of simultaneous linear equations in three unknowns, together with an introduction about the method of Newton's polynomial interpolation. It is not necessary to discuss the concept of rank in relation to the augmented matrix in Gaussian elimination. Instruction can cover the meaning of a linear combination of plane vectors to the extent adequate to explain the conditions to determine if a system of linear equations that has a unique solution, infinite many solutions or no solution, yet without extending to the concept of linear independence. The discussion can also extend conceptually to solving a system of linear equations involving more unknowns, which is practically solved by way of computer software. Arrangement should be made during instruction to demonstrate how to use conveniently accessible		g-V-4 a-V-3

		computer software to solve a system of linear equations. (The algebraic criterion to determine the geometrical relationship among three planes can be assessed after instruction but not in a formal setting. ★)		
A-11A-3	**Matrix operations:** Basic concepts of matrices should include the definition of a matrix, scalar multiplication, addition and subtraction of matrices, products of matrices and inverse matrix. Coverage should also include treating data tables as matrices and demonstrating how to use software to carry out matrix operations.	It is possible to conceptually discuss the inverse of a matrix of order n. For actual computation, however, it is only necessary to find the inverse of a matrix of order 2.		a-V-3
A-11A-4	**Laws of logarithms:** Instruction can begin with the concept of 10^x and the laws of exponentials to help students recognize the laws of common logarithm, their basic applications, together with their use to solve exponential equations.	Students are expected to recognize the concept of a logarithm of arbitrary base. Yet no extensive practices are required.	Calculator	a-V-1 n-V-2
F-11A-1	**Graphs of trigonometric functions:** Coverage should include the graphs of the sin, cos and tan functions, their domains and ranges, periodicity and the mathematical models of periodic phenomena. (Notice that the definitions and graphs of cot, sec and csc can be introduced in classes as extended topics, but they should not be assessed in public examinations. ※)		Grid paper, calculator	f-V-3 n-V-7 g-V-2
F-11A-2	**Superimposition of sine and cosine functions:** Coverage should include the frequency and amplitude of superimposed waves that have the same		Grid paper, calculator	f-V-3 s-V-1

	frequency.			
F-11A-3	**Application of matrices:** Coverage should include linear transformations on a plane and the second order transition matrices.			f-V-5 a-V-3
F-11A-4	**Exponential and logarithmic functions:** Coverage should include exponential functions and their graphs, mathematical models of phenomena with proportional growth or decay, graphs of common logarithmic functions, as well as their applications in sciences and finance.	Students are expected to recognize the concept of a logarithmic function of arbitrary base. The main focus is to help students understand that a logarithm of arbitrary base can be transformed into a common logarithm. It is not necessary to deal with problems with logarithms of various bases presented within a single equation. Students should realize that any exponential functions of the form a^x can be rewritten in the form of 10^{kx}, where $0 < a \neq 1$.	Grid paper, calculator	f-V-4 g-V-2
D-11A-1	**Subjective and objective probability:** Coverage should include using the properties of probability to examine the rationality of the subjective probability of an event, as well as obtaining the objective probability of an event from given data.		Calculator	d-V-3 d-V-5
D-11A-2	**Conditional probability:** Instruction should cover the meaning and application of conditional probability, as well as the meaning and application of the independence of events.			d-V-3
D-11A-3	**Bayes theorem:** Instruction should cover the multiplication formula of conditional probability, and the Bayes theorem together with its application.			d-V-3
Grade 11 (B strand)				
N-11B-1	**Radian measure:** Coverage should include the definition of radian		Calculator	n-V-7

	measures, how to find arc lengths and areas of sectors, as well as the operation of the rad function key on a calculator.			
S-11B-1	**The concept of space:** Coverage should include the basic properties of space, the relationship between two straight lines, between two planes, between a straight line and a plane with respect to their spatial positions. Instruction should include a discussion about the distance between two points on the net of a cuboid and also an introduction to the concepts of longitude and latitude on a sphere.	Instruction should attend to students' needs in geography classes and help them recognize the meanings of a big circle and a small circle on a sphere, the perpendicular relationship between a straight line and a plane, the parallel and perpendicular relationship between two straight lines, as well as the perpendicular relationship between two planes. Instruction should also include a discussion about the concept of a dihedral angle. However, it is not necessary to geometrically deal with general dihedral angles other than the case for the right angles.		s-V-2
S-11B-2	**Conic sections:** Instruction should help students recognize the curves of conic sections by visually inspecting the section of the conic surface formed by passing a cutting plane through a right cone at various positions, as well as their manifestations in nature.		Conic section model	s-V-2
G-11B-1	**Plane vector:** Coverage should include the addition, subtraction and scalar product of vectors in a plane coordinate system as well as the concept of a linear combination of vectors.			g-V-1
G-11B-2	**Operations of plane vectors:** Coverage should include orthogonal projection, inner product of vectors, criteria to			g-V-5

	determine if two vectors are parallel or perpendicular, and the angle between two vectors.			
G-11B-3	**Proportion on a plane:** This entry is concerned with problems of proportion that are observed in everyday life and in plane geometry (in relation to design and perspective drawing).			g-V-4
G-11B-4	**Spatial coordinate system:** Coverage should include point coordinates, the distance between two points, and the projections from a point to the coordinate axes or to the coordinate planes.	Space coordinates can be computed by means of the longitudes and latitudes on a sphere the center of which is positioned at the origin.		g-V-1
A-11B-1	**Matrix and data table:** Coverage should include a discussion about the meaning of multiplying a matrix with a vector in terms of linear combination; the meaning of a system of linear equations in two unknowns; matrix addition, subtraction and multiplication as well as the inverse of a second order matrix. Examples should be provided that treat matrices as data tables and then demonstrate how to use computer software to carry out matrix operations.			a-V-3
F-11B-1	**Mathematical model of periodic phenomena:** Coverage should include the graphs of sine functions and their periodicity, their amplitudes, periods and frequencies. Adequate examples of periodic phenomena should be provided during instruction.		Graph paper, calculator	f-V-3 n-V-7
F-11B-2	**Proportional growth model:** Coverage should include exponential function and logarithmic		Graph paper, calculator	f-V-4 n-V-2

	function and their applications in daily life, such as in earthquake magnitude, finance and management, average growth rate, continuous compound interest together with an introduction to the Euler's constant e, and the natural logarithmic function.			
D-11B-1	**Subjective and objective probability:** Coverage should include examining the rationality of subjective probability based on the properties of probability, and an estimation of subjective probability based on given data.		Calculator	d-V-3 d-V-5
D-11B-2	**Uncertainty:** Coverage should include conditional probability, Bayes' theorem, the concept of independent events and its application, and the relationship between contingency tables and Venn diagrams.			d-V-3
Grade 12 (Broadened and Deepen Elective Mathematics I)				
N-12 I-1	**Limit of sequence:** Coverage includes the limit of a sequence, computational properties of limit, and the squeeze theorem. Instructors should help students recognize the Euler's constant e based on the concept of compound interest.	Instruction should include Newton's method for finding roots, demonstration regarding how to use calculators to find the limit of an indeterministic sequence of values that resulted from the root finding process, and the use of the location of roots theorem to find a suitable starting value for Newton's method. Coverage should also include using the squeeze theorem to demonstrate how to estimate the value of π, and help students to realize the sandwiching phenomena by using the estimated values from a calculator. (※ Introduction to the natural	Calculator	n-V-8 n-V-2

		exponential function and the natural logarithmic function can follow after the introduction of the constant e.)		
N-12 I -2	**Infinite geometric series:** Coverage should include periodic decimal expansion and the summation symbol Σ.			n-V-8
N-12 I -3	**Complex number:** Coverage should include the complex plane, polar form of a complex number, the four operations with complex numbers, the absolute value of a complex number and its geometric meaning, de Moivre's theorem, and the nth root of a complex number.			n-V-3 n-V-4 g-V-4 s-V-1
G-12 I -1	**Curves of the second degree:** Coverage should include the equations of a parabola, ellipse and hyperbola in the standard form, and the parametric form of an ellipse.	Instruction should include the concepts of translation and dilation, as well as the standard formula of a rotated ellipse (using the origin as the center). Students should recognize the xy term of a quadratic equation with two unknowns from the standard formula of an ellipse that has been rotated to a slanting orientation. However, it is not necessary to deal problems involving the xy term any further. Coverage can also include an extension to the parametric equations of a circle from the parametric equations of an ellipse.		g-V-4 g-V-5
A-12 I -1	**Complex number and equation:** Coverage should include imaginary roots of equations, fundamental theorem of algebra, and the property of imaginary roots occurring in conjugate pairs in equations with real coefficients.			a-V-2 n-V-3
F-12 I -1	# Function: Coverage	Students should realize that		f-V-1

	should include correspondence relationship, the symmetry relation of odd and even functions as shown in their graphs, the meaning of convexity and concavity, functional derivation of inverse functions together with the symmetric relationship of the graphs of a function and its inverse function with respect to the straight line $y = x$,and composite functions. #	functions can be operated on as objects within the context of learning calculus and related topics and be competent with these operations. Examples includes $f \pm g$ and $f \circ g$.		g-V-2
F-12 I -2	**Limit of function:** Coverage should include the continuity of function and the limit of a function at the real number a, the properties of operations on limits, absolute value function and piecewise defined function, the intermediate value theorem, and the squeeze theorem.	This entry should be connected with the foundational topic on division of polynomials covered in the tenth grade. The purpose of this entry is to deal with differentiation and should not be over extended in instruction.	Calculator	f-V-6 n-V-2 a-V-1
F-12 I -3	**Differentiation:** Coverage should include the limit definitions of derivative and function, tangent and derivative, derivatives of polynomial functions and simple algebraic functions, basic formula of differentiation including those for the scalar product and the properties of addition and subtraction.	※ The derivatives of functions such as $\sin x$, $\cos x$, 2^x and 3^x can be used as examples for differentiation.		f-V-6 n-V-7 a-V-2
F-12 I -4	**Derivative as a function:** Coverage should include the product rule, division rule, chain rule, higher order derivative, Leibniz's notation, determination of the monotonicity and convexity of functions, first order approximation and basic optimization problems.	This entry will focus on using polynomial functions as the examples for taking derivatives. Chain rule will be applied chiefly on polynomials of the form $(x-a)^n$. The Taylor expansion of polynomial functions will also be covered in this entry.		f-V-7 f-V-2
F-12 I -5	**Riemann sum:** Coverage should include the connection between Riemann sum and definite		Calculator	f-V-9 n-V-8

	integral.			
F-12 I -6	**Integration:** Coverage should include the anti-derivative of polynomial functions and indefinite integral, the meaning of definite integral with respect to area, displacement, and the net change, and also the fundamental theorem of calculus.	This entry does not concern with the integration by part nor the integration by substitution. The instruction of definite integral will focus mainly on polynomial functions, but can extend to other functions or given figures when the meaning of area is fairly clear. Examples can also include continuous piecewise functions with two to three segments, as well as applying absolute value to linear or quadratic functions.		f-V-8 f-V-2
F-12 I -7	**Applications of integration:** Coverage should include the average of the values of a continuous function, area of a circle, volume of a sphere, integration by dissection, and the volume of a revolving body.			f-V-9
D-12 I -1	**Discrete random variable:** Coverage should include the concepts of expected value, variance and standard deviation, the concept of independence, Bernoulli trials and replications of experiments.			d-V-4
D-12 I -2	**Binomial and geometric distributions:** Coverage should include the properties and parameters of the binomial distribution and the geometric distribution.	Instruction should include applications to checking the rationality of the probabilities of the occurrence of given events.		d-V-4 d-V-5 a-V-1
Grade 12 (Extended elective Math II)				
N-12II-1	**Complex number:** Coverage should include the complex plane, the four operations of complex numbers and their absolute values.			n-V-3
N-12II-2	**Infinite geometric series:**			n-V-8

	Coverage should include periodic decimal expansion and the summation symbol Σ.			
A-12II-1	**Linear programming:** Coverage should include the extreme values of the linear objective function and the system of parallel lines.			a-V-4
A-12II-2	**Imaginary roots of equation:** Coverage should include imaginary roots of equations, fundamental theorem of algebra, and the property of imaginary roots occurring in conjugate pairs in equations with real coefficients.			a-V-2 n-V-3
F-12II-1	**＃ Function:** Coverage should include the correspondence relationship, the symmetry relation of odd and even functions as shown in their graphs, and the meaning of convexity and concavity. ＃	Students should realize that functions can be operated on as objects within the context of learning calculus and related topics. Examples includes $f \pm g$ and $f \circ g$.		f-V-1 g-V-2
F-12II-2	**Limit of function:** Coverage should include enabling students to understand the continuity of function and the limit of a function at the real number a, the properties of operations on limits, the intermediate value theorem, and the squeeze theorem.	This entry should be connected with the foundational topic on division of polynomials covered in the tenth grade. The purpose of this entry is to deal with differentiation and should not be over extended in instruction.	Calculator	f-V-6 n-V-2 a-V-1
F-12II-3	**Differentiation:** Coverage should include the limit definitions of derivatives and derivative of a function, tangent and derivative, derivatives of polynomial functions, basic formula of differentiation including those for the scalar product and the properties of addition and subtraction			f-V-6 n-V-7 a-V-2
F-12II-4	**Derivative of a function:** Instruction should include second derivative, Leibniz's			f-V-7 f-V-2

	notation, criteria for determining the monotonicity, concavity and convexity of functions, application to optimization problems, and the marginal meaning of derivative in economics.			
F-12II-5	**Integration:** Coverage should include inverse functions and integration of linear and quadratic functions. the meaning of definite integral with respect to area and the net change, as well as the fundamental theorem of calculus.	This entry does not concern with the integration by part and the integration by substitution. Examples for integration can be extended to other functions or given figures when their meaning of area is fairly clear.		f-V-8 f-V-2
F-12II-6	Application of integration: Coverage should include the concepts of taking averages of continuous functions, and the meaning of total and residual.			f-V-9
D-12II-1	**Discrete random variable:** Coverage includes the concepts of expected value, variance and standard deviation, the concept of independence, Bernoulli trial and the replication of experiments.			d-V-4
D-12II-2	**Binomial distribution:** Coverage should include the properties and parameters of the binomial distribution.	The focus of this entry is to check the reasonableness of the probability of a given event.		d-V-4 d-V-5 a-V-1

VI. Implementation Directions

1. Curriculum Development

(1) Mathematics is a form of language. It is one of the foundations and a way of thinking that facilitates deeper learning. The education system should strive to provide equal opportunities for everyone to learn mathematics. The organization of instruction should emphasize arrangement in a progressive manner and develop individualized instruction at the right time. Issues related to gender equality, human rights, environmental and marine

education can also be integrated into the development of the mathematics curriculum whenever appropriate. These efforts can help nurture students' core literacy and enrich their studying of mathematics.

(2) Within the specifications of the flexible curriculum policy, instructors at the elementary and junior high school levels should make instructional plans that help students build up their foundation in mathematics and engage them in inquiry activities, thereby providing them opportunities to explore, discuss and foster their interest in mathematics. In such activities, however, students should have gained the prerequisite knowledge prior to learning specific topics. It is hoped that every student can experience meaningful learning accordingly.

(3) The curriculum development can be adjusted according to the needs of the students. The pace of instruction can be slackened for slow-learning students, whereas broadened and deepened curriculum or special topics can be designed for those who accelerate in their learning. As for those who are left behind, remedial courses should be planned to assist them in a timely manner.

2. Teaching Material Selection and Composition

(1) The compilation of textbooks should abide by the rationales, curriculum goals, core literacies and learning focuses as delineated in these curriculum guidelines. It is expected that high quality textbooks be compiled that help the students to pursue reading on their own and the instructors to carry out their instructional practices.

(2) Textbook compilation should be accompanied by the compilation of instructors' manuals. The purpose of these manuals is to enable the instructors to gain deeper understanding of the teaching materials and the curriculum. It is hoped that the manuals can enhance the teaching efficacies of the instructors so that they can attend to students at different levels and meet their different needs.

(3) The Learning Contents are organized according to the idea of presenting specific groups of mathematical concepts in a clear manner. The arrangement should not be so interpreted that each entry corresponding to a teaching unit in a strict sense. It is not necessary for textbooks to be compiled according to the serial order of the topics within any grade. For some entries in relation to Grades 7-12, there are three special symbols, namely, ※, ★ and ＃. Textbook authors must pay careful attention to their meaning while compiling textbooks. These symbols should be marked at their designated places and their meaning clearly explained in the textbooks.

(4) Calculators are to be formally introduced in the junior high school for the first time with this curriculum. It is suggested that textbook authors consider devoting specific sections to deal with the properties of various operations of the calculators with concrete examples as well as discussing mistakes that can be easily made. The teaching materials should make it clear to students that the results from calculators and computers may carry errors or differences due to the limitation of the number of significant figures in answers, and also errors caused by students' improper operation on calculators. Some examples of such problems include data entry errors, procedural errors and insufficient number of significant figures. Calculators for junior high school students should have the capacity to handle the four arithmetic operations with whole numbers, floating-point numbers and numbers in scientific notations. They should have the functions to handle percentages, square roots and taking the sums and averages of given sets of data. Calculators for junior high school students should have the further capacity to handle step-by-step operations involving powers, trigonometric and inverse trigonometric functions, exponential and logarithmic functions, and finding the variances and standard deviations of given sets of data. Instructors' manuals should make it a point to the instructors to allow their students to have actual experiences of plotting curves before turning to the graphing functions of some computer software, and guide them to observe the characteristics of the graphs as well as to explain what those characteristics mean.

(5) While compiling, authors should pay attention to the organic coherence of the contents in the structure of the textbook. The materials should be able to reflect the inter-connectedness of the mathematical concepts being presented. Furthermore, the selected materials should fit the mathematical topics well and allow for the connection and application of mathematics to other disciplines as well as problems encountered in daily life.

(6) As regards the style of presentation, the topics should be organized in an orderly and progressive manner, with an appropriate level of delineation, multiple representations, cues that can raise learning motivation, and being mindful of the possible status of students' psychology of learning. Besides, a balance should be maintained in keeping the presentation as appealing to both the intuitive and rigorous reasoning in the minds of students. Authors should be aware of the necessity to present mathematical properties or arguments first with specific examples before using general or abstract reasoning. As regards the wording, they should be written with appropriate expressions and in accordance

with the reading levels of the students. For students who will enter junior high schools, they may need to adapt to new learning environments and adopt new methods of learning. Textbooks for these students must pay special attention in the arrangement of learning materials and the use of suitable teaching methods in order to facilitate the instructors in enabling their students to bridge over the differences in learning conditions across different stages.

(7) Textbooks should provide enough learning tasks and exercises to reinforce students' learning. These tasks should be conducive to meaningful learning and develop students' abilities in mathematical thinking. It is hoped that instructors can find out in real-time the learning status of their students through observing their performances in class exercises or from formative assessments, and can adjust their instructional activities accordingly. The exercises at the end of each section should match well with the topics introduced, can stimulate students into deeper thoughts and have problems arranged in an ascending order of difficulty. Authors should avoid posing technical problems or learning tasks that lack substantive meaning, remotely related to the contents or involving contexts that do not make sense to students.

(8) While designing learning tasks and exercises, textbook authors should pay attention to connect, when appropriate, these activities with the daily experiences of students, the contents of other disciplines or subjects, as well as issues related to gender equality, human rights, environmental and marine education. For example, learning tasks can be set within the context of gender equality or issues related to sustainable development. For those occasions where role play activities are appropriate, all genders should be offered equal opportunities to participate.

(9) Textbook authors should select suitable materials from the history of mathematics, ethnomathematics and biographies of mathematicians as prompts to arouse students' interest, nurture their appreciation for the development of mathematics, and realize the contributions and achievements made by different cultural groups and genders. Schools with special indigenous focus are encouraged to compile their own teaching materials that can accommodate local cultural interests, thereby facilitating the instructors to implement culturally responsive teaching.

(10) Reviewers of textbooks should be aware of the spirit of the curriculum guidelines and proceed in their review by referring to the above-mentioned guiding principles for the

compilation of textbooks.

(11) Mathematics instructors should select suitable textbooks for their students. Whenever necessary, instructors can compile their own teaching materials.

(12) For instructors who would prefer to compile their own teaching materials, they should base their compilation on the specifications of the curriculum guidelines and comprehend the spirit therein. They should, however, avoid overextending the coverage that may cause unnecessary frustration in learning for students. In addition, instructors can compile challenging and thoughtful materials for those students who are gifted in mathematics.

3. Teaching Implementation

(1) The learning focuses are set with respect to the kinds of mathematical literacy that students are expected to have at the end of each stage or grade. Instructors should design different lesson plans according to the levels of their students by referring to the learning focuses together with their descriptions and remarks. The selection of content materials should pay attention to the local habitat as well as the actual daily experiences of the students. The subject matters should be interesting to students. Instructors may also want to monitor the quality of the learning environment. All these measures are expected to facilitate the instructors in their instructional duties.

(2) Notice that the learning focuses are discrete, itemized entries, yet both instruction and learning are continuous processes. The specifications for each stage or grade reflect the main points that should be covered in lessons and learned by students. Instructors can, based on their experience, organize the materials so that they can prepare students to learn further topics at a later grade or a later stage, or to receive remedial type of instruction.

(3) Classroom instruction should be student-centered, with developing students' interest and abilities in mathematics as its purpose. Instructors can encourage students to provide multiple solutions to problems and exchange their ideas with other students. Since students have different learning paces, instructors should avoid teaching the whole class as if every student has the same learning pace. Instead, instructors should analyze the readiness of students and design individualized teaching and assessment so that appropriate diagnosis, guidance and assistance can be provided to every student in due course.

(4) In the formulation of the Learning Content, no teaching methods have been stipulated in advance. Instructors can choose methods that are sufficiently meaningful to students and adapt to students' current needs based on their ages, their prior knowledge, the nature of

the subject matters and the actual situation at the teaching site. Some effective teaching methods include cooperative learning and inquiry.

(5) While designing instructional activities, instructors should be aware that students at different learning stages may have different learning styles. The instructional activities should correspond well to the instructional objectives. Instructors should encourage and guide their students to engage in mathematical inquiry and in cooperative problem solving.

(6) Mathematics instruction should pay attention to helping students understand the connection among the concepts of number, quantity and shape. Students should have opportunities to become fluent in using these concepts as well as their inter-relationships through hands-on activities, direct measurements and intuitive reasoning. Their understanding in these areas should gradually change from abstract concepts into concise and effective mathematical language. By undergoing further reflection, argumentation, practices and problem-solving activities, students can gradually establish firm understanding and mastery of these concepts and use them as steppingstones to learn other concepts.

(7) Instructors can use guidance, enlightenment and direct teaching to enable students, while working on problems from real life contexts, to use their prior mathematical knowledge as a basis for formulating new mathematical concepts that are required to solve the problems and to strategically choose correct and efficient problem-solving procedures. Instructors can provide inspiring, critical and real-life applications to students to stimulate ideas from them. However, instructors are advised not to provide open-ended problems that are worthless or meaningless. They should also avoid revealing prematurely the problem-solving methods and the answers.

(8) Instructors should guide students to experience for themselves the process of making connection between mathematics and daily life situations. Furthermore, instructors should nurture their students to form the habit of observing the world around them from a mathematical viewpoint, and also to recognize the mathematical meaning, characteristics and relationships behind a given problem. Students should form the habit of representing given problems into mathematical ones and then try to solve them, thereby enhancing their ability to apply mathematical knowledge in context. While developing students' ability to formulate problem solving strategies, instructors should help students gain deeper understanding of mathematical concepts. For example, instructors can make good use of issues from gender equality, human rights, environment and marine education as contexts

for problems. Not only will students gain a deeper understanding of the mathematical concepts while developing their problem solving strategies, their sense of self-identity and international perspectives will also be enhanced.

(9) When learning mathematics, students should move back and forth between solving application problems from daily life and engaging in abstract and formal reasoning in a coherent manner. This is necessary if their mathematical abilities are to develop evenly. It is not necessary for instructors to stress too much on problems posed within some real-life contexts. This will interfere with students' development of abstract and formal thinking abilities. Nor should instructors overemphasize the learning of abstract thinking which will hinder students' ability to apply mathematics to solve problems in everyday life.

(10) Mathematics differ from other disciplines in that it has a nested layers of knowledge structure and its development relies on both intuition and reasoning. Instructors should regard mistakes made by students as part of their learning processes, and diagnose the causes of their problems (for example, ideas not communicated, liberal extension of agreed upon conventions, errors in reasoning, etc.), and help them face their problems. Instructors should provide enough time and encourage their students to explain their reasons and ideas, affirm the validity of good thinking or use critical examples to clarify misunderstandings.

(11) Learning well in mathematics relies on the students' ability to integrate related concepts into a networked sense of wholeness or intuition for each topic they learn, which will form the basis for learning the next topic. Gaining a sense of overall self-confidence depends on the students' proficiency in carrying out certain procedures (such as computational techniques, problem-solving techniques, etc.). This kind of proficiency hinges on the instructors' being able to assign illuminating exercises to students, rather than drilling of the routine type. This will allow students the opportunity to accomodate their newly learned concepts and be able to assimilate them with their former network of mathematical knowledge

(12) In order to implement the objective of guiding every student to excel in learning, instructors should, accordingly, try to help all students learn mathematics well. They can teach according to the students' learning performances and their needs. Remedial teaching should be implemented in a timely manner for students who are behind and cannot achieve the objectives that are expected for their stages. For students who are quick in their learning, advanced elective courses should be provided to stimulate their enthusiasm for deeper

learning.

(13) In order to nurture students' ability to use calculators, all students in junior high school and upper secondary school should own or have access to standard calculators. Instructors should abide by the content of the mathematics curriculum and teach their students the correct methods and attitudes in using calculators

(14) Instructors can introduce appropriate amounts of knowledge from the history of mathematics, ethnomathematics and biographies of mathematicians. They can infuse into their lessons humanistic views of mathematics and nurture in students a sense of appreciation of the progress in mathematics. However, these contents should not be assessed formally.

(15) By making use of the mathematical background of students and inquiry activities, instructors can encourage students to apply mathematical knowledge to solve practical problems in daily life. They can collaborate with instructors from other disciplines or domains and develop examples of real application of mathematics in different disciplines. These efforts can help students build up the competence to apply mathematics in other disciplines or domains.

4. Teaching Resources

(1) Instructors should make appropriate use of teaching aids when they teach so as to help students understand the concepts visually and in thoughts, thereby raising their teaching effectiveness.

(2) It is preferable that teaching aids are self-made. Simple teaching devices can be designed by the instructors and students together using materials on hand in a flexible manner, whereas more complicated teaching aids should be provided by schools or by competent authorities. Whenever necessary, a room can be set up to store the teaching aids. For teaching aids that are used frequently, including rulers, protractors, compasses and calculators, it is more convenient for students to purchase them on their own and keep them for long-term use. Instructors can use high-end function calculators and programmable calculators depending on the situation. To help their students visualize concrete images that can reflect abstract concepts, instructors can flexibly use computers to assist their instruction. For examples, instructors can use computers to teach the following topics: graphs of functions, solid geometry, solving equations and statistics.

(3) Mathematics is a science of patterns. While students engage themselves in mathematical

inquiries, oftentimes they must manipulate large numerical quantities and big data (basic vocabulary of mathematical language). They should thus possess the ability to operate calculators to carry out mathematical inquiry activities. When teaching students how to use calculators, instructors should pay attention to fostering students' correct attitudes towards using calculators. Students should be aware that numerical calculations from calculators and computers may carry certain errors due to the limitation of the number of significant digits in the answers. Thus, in applications, they should consider the tolerance limits of this drawback in using calculators. In addition, instructors should help students understand the errors that may occur when operating calculators, such as data entry errors, programming errors and insufficient significant digits. With the above-mentioned limitations in mind, students can use calculators to solve problems or assist in verifying answers. They can combine the use of calculators with mental calculation and estimation to check the reasonableness of the calculation results, thereby strengthening their number sense. More specifically, after students become proficient in the principles of calculation, they can consider using calculators both in learning and in assessment activities so as to avoid too many complicated calculations that may reduce their learning efficiency. They can use calculators to handle problems involving statistical data, exponents, logarithms and trigonometric ratios. Instructors should let students have the actual experience of plotting curves before turning to the graphing functions of some computer software. They can guide students to observe the features of various graphs of functions that are produced by computer graphics and explain their meaning afterwards.

5. Learning Assessment

(1) Assessment is the process of evaluating the effectiveness of teaching and learning. Instructors should try to improve their teaching through various kinds of assessment, such as paper-and-pencil tests, performance assessment, discussions, oral responses, observations, homework, projects or group reports, etc. Instructors should choose appropriate modes of assessment depending on the needs of the occasion.

(2) In addition to summative assessment, instructors should use formative assessment to find out the status regarding their students' learning performances, learning difficulties and whether there are gaps between their current levels of understanding and the learning goals. They should provide immediate feedback to students and modify their teaching strategies to facilitate students' learning.

(3) Learning assessment activities should cover learning achievements, learning readiness, learning motivation and the learning processes of students. Analyses can then be done to evaluate whether students have met the requirements of the major learning objectives. Instructors should set up assessment criteria based on the content of the textbooks, teaching goals and the main ideas of the relevant topics. Instructors should not pose very difficult problems in learning assessment. This is because learning assessment is not a norm-referenced test, and so it should not emphasize the discrimination of abilities within the whole class or the whole school.

(4) By analyzing the assessment results of individual students, instructors can understand what prior knowledge and experiences they may have, trace back from their mistakes to their learning problems and then guide them to clear up their misunderstandings. The assessment results of the whole class can serve as feedback to the instructors so that they can improve their instructional practices. As regards the assessment results of the whole school or results from public examinations, school officials and instructors can provide them as feedback or as reference information to those who will oversee future curriculum revisions.

(5) Instructors should pay attention to the timing of the assessments so as to avoid making wrong or inappropriate interpretations of the assessment results. Assessment of students' performances at the beginning of a topic can be used as the basis for drawing up teaching plans. Assessment during the learning process can identify students' learning difficulties so that remedial teaching can be implemented in a timely manner. Assessment after the learning process can serve as feedback with respect to students' learning and as a reference regarding how to guide students subsequently.

(6) Assessment activities should fulfill the purposes of the assessment. The problems selected should adequately reflect students' learning status, and can substantiate the functional characteristics of the specific assessment method adopted. Sufficient time should be allowed for students to think through the assessment tasks. Moreover, true-or-false and multiple-choice problems that may easily lead to guessing should be avoided. Instructors should ask their students to explain their answers in detail. This will allow them to understand the thinking process of their students, and can also serve as a basis for setting up the criteria for analytical scoring. According to the extent of the problem being solved, partial credits can be assigned and students should be informed of what are being incorrect in their answers.

(7) The assessment of academic achievement, competence test and admissions test should all align with the curriculum guidelines. In order to foster a healthy learning culture and achieve the goal of helping students to develop their mathematical thinking and mathematical literacy, assessment should allow students to have sufficient time to take the tests and avoid those elements that may promote students' rote learning. The upper secondary school curriculum separates into two different strands starting from Grade 11. The purpose of this arrangement is to provide students who have different needs of mathematics to have the chance to choose an appropriate mathematics course for themselves. College admission tests should adjust their content accordingly.

(8) In order to foster in students the correct attitude towards using mathematical tools, not only should the use of calculators be integrated into the lesson plans, they should be allowed to be used in achievement, competence and admissions tests along with the use of other tools such as rulers, set squares, protractors and compasses. The implementation of this policy would help the students to correctly use the tools as a habit. When posing problems for assessment purposes, schematic diagrams can be presented in addition to the item stem. But they should be explicitly labeled as schematic diagrams.